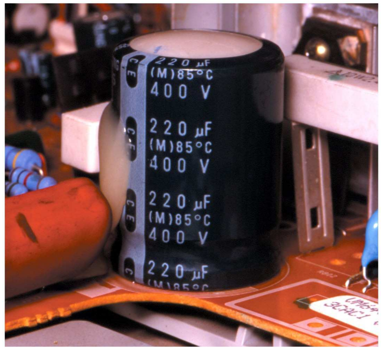
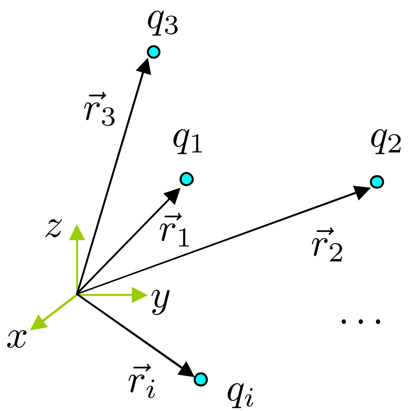
Energia electrostàtica, capacitat i dielèctrics

Energia potencial electrostàtica

Emmagatzemada a condensadors



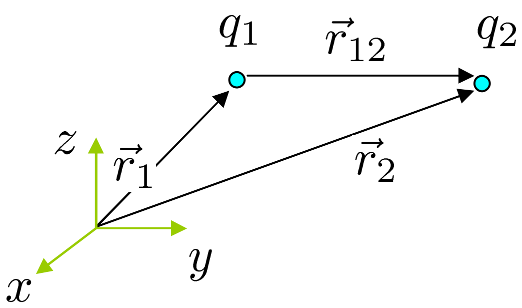
Sistema de càrregues puntuals



The electrostatic potential energy of a system of point charges is the work needed to bring the charges from an infinite separation to their final positions.

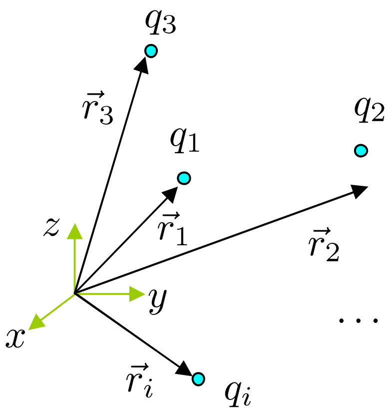
positiva o negativa

ELECTROSTATIC POTENTIAL ENERGY OF A SYSTEM



$$U = q_1 V_1 = q_2 V_2 = \frac{1}{2}\left(q_1 V_1 + q_2 V_2\right)$$

$$U = k\frac{q_1 q_2}{r_{12}}$$

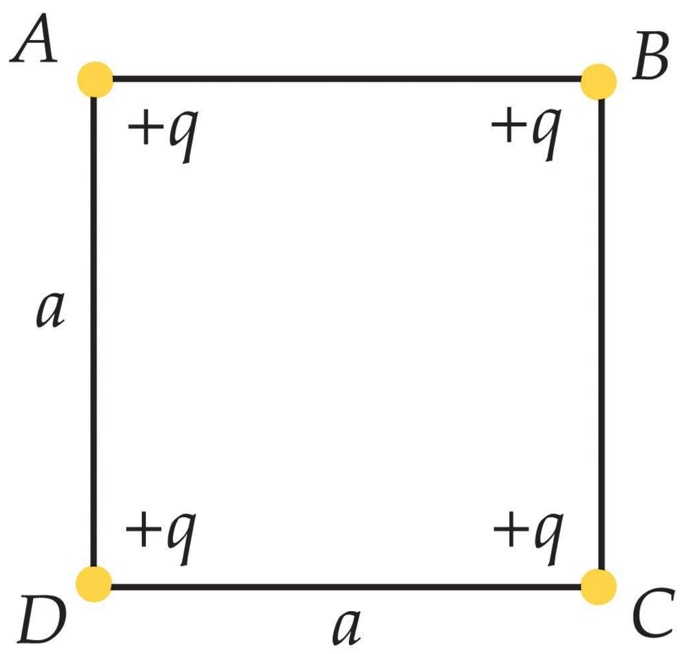


$$U = k \sum_{\substack{i<j=1 \\ \text{parelles}}}^{N} \frac{q_i q_j}{r_{ij}}$$

$$= k\left(\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \ldots + \frac{q_2 q_3}{r_{23}} + \ldots + \frac{q_3 q_4}{r_{23}} + \ldots\right)$$

$$= \frac{1}{2} k \sum_{i \neq j=1}^{N} \frac{q_i q_j}{r_{ij}} = \frac{1}{2} \sum_{i=1}^{N} q_i V_i$$

$$V_i = k \sum_{\substack{j=1 \\ j \neq i (\text{fixed})}}^{N} \frac{q_j}{r_{ij}}$$

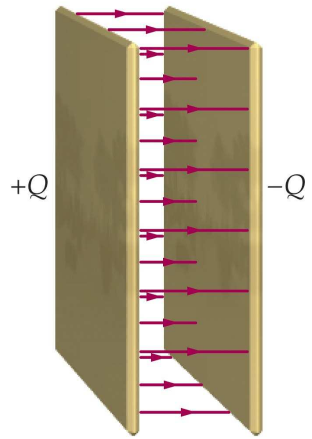


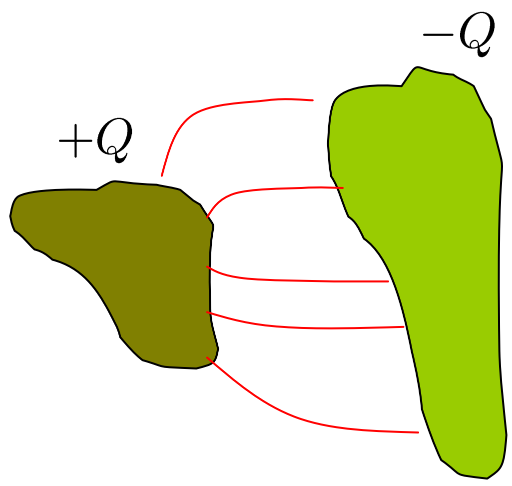
$$q = 1\,\mathrm{C}$$

$$a = 1\,\mathrm{m}$$

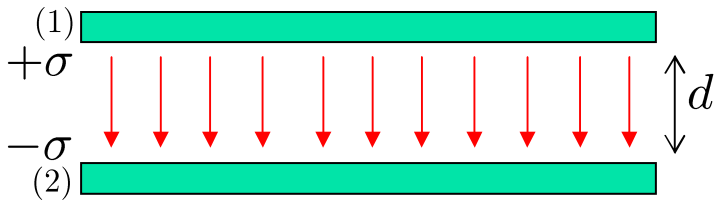
$$U = (4 + \sqrt{2})\frac{kq^2}{a} = 4.87\ 10^{10}\ \mathrm{J}$$

Condensador (capacitor)





De cares planes i paral.leles



$$E = \frac{\sigma}{\epsilon_0} \quad \text{entre plaques}$$

$$V = \frac{W}{q}(2 \to 1) = V_1 - V_2 = -\int \vec{E} \cdot d\vec{\ell} = Ed$$

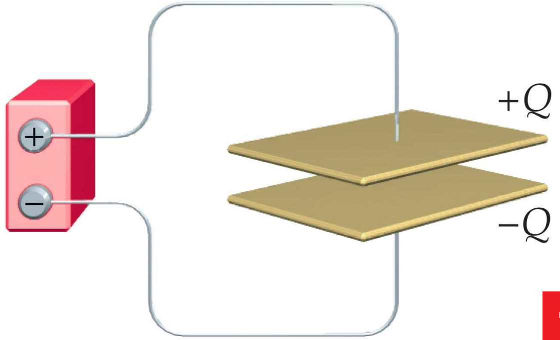
$$V = \frac{\sigma d}{\epsilon_0} = \frac{d}{\epsilon_0 A} Q$$

linealitat
superposició

àrea

capacitat $C = \dfrac{Q}{V}$ $Q = CV$

De cares planes i paral.leles $V = \dfrac{d}{\epsilon_0 A} Q \Rightarrow C = \dfrac{\epsilon_0 A}{d}$



Unitats $[C] = \dfrac{[Q]}{[V]} = \dfrac{\mathrm{C}}{\mathrm{V}} = \mathrm{F}$ farad

submúltiples: $\mathrm{mF},\ \mu\mathrm{F},\ \mathrm{pF},\ \mathrm{fF}$

Table 3. SI prefixes

Factor	Prefix	Symbol	Factor	Prefix	Symbol
10^{24}	yotta	Y	10^{-1}	deci	d
10^{21}	zetta	Z	10^{-2}	centi	c
10^{18}	exa	E	10^{-3}	milli	m
10^{15}	peta	P	10^{-6}	micro	μ
10^{12}	tera	T	10^{-9}	nano	n
10^{9}	giga	G	10^{-12}	pico	p
10^{6}	mega	M	10^{-15}	femto	f
10^{3}	kilo	k	10^{-18}	atto	a
10^{2}	hecto	h	10^{-21}	zepto	z
10^{1}	deka	da	10^{-24}	yocto	y

$$\epsilon_0 = \frac{1}{36\pi\ 10^9}\frac{\mathrm{C}^2}{\mathrm{Nm}^2} = \frac{1}{36\pi\ 10^9}\frac{\mathrm{F}}{\mathrm{m}}$$

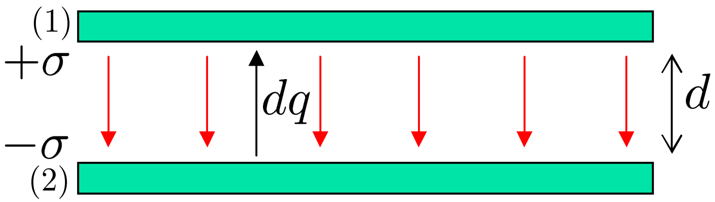
Quina és les capacitat d'un condensador de cares planes paral.leles de 1 cm^2 d'àrea i 1 mm de separació?

$$C = \frac{\epsilon_0 A}{d} = \frac{\frac{1}{36\pi}\ 10^{-9}\,\frac{\mathrm{F}}{\mathrm{m}}\ 10^{-4}\mathrm{m}^2}{10^{-3}\mathrm{m}} = 0.88\ 10^{-12}\,\mathrm{F} \approx 1\ \mathrm{pF}$$

Capacitat d'una esfera conductora?

$$V = V_{\mathrm{esfera}} - 0 = \frac{Q}{4\pi\epsilon_0 R} \quad\Longrightarrow\quad C = 4\pi\epsilon_0 R$$

Energia potencial electrostàtica d'un condensador



Procés de càrrega

$$q \longrightarrow Q$$
$$v \longrightarrow V$$

$$v = \frac{q}{C}$$
$$dU = v dq$$

$$U = \int dU = \int v\, dq = \int_0^Q \frac{q}{C} dq = \frac{Q^2}{2C} = \frac{1}{2} C V^2$$

Vàlid condensador general

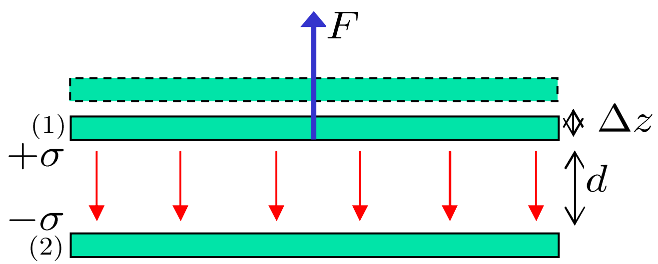
Càlcul de forces amb la llei de conservació de l'energia

$$\left.\begin{array}{r} \text{força } F \\ \text{desplaçament } \Delta z \end{array}\right\} \qquad F\Delta z = \Delta U$$

$$\text{Si } \Delta U < 0 \Rightarrow F < 0$$ Si l'energia baixa el treball es fa espontàniament

$$\text{Si } \Delta U > 0 \Rightarrow F > 0$$ Si l'energia puja s'ha de forçar

Exemple

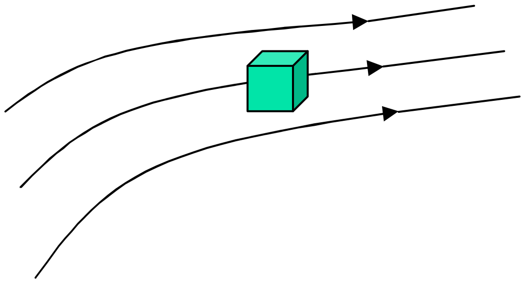


$$\Delta U = \frac{Q^2}{2C_{\text{final}}} - \frac{Q^2}{2C_{\text{inicial}}}$$

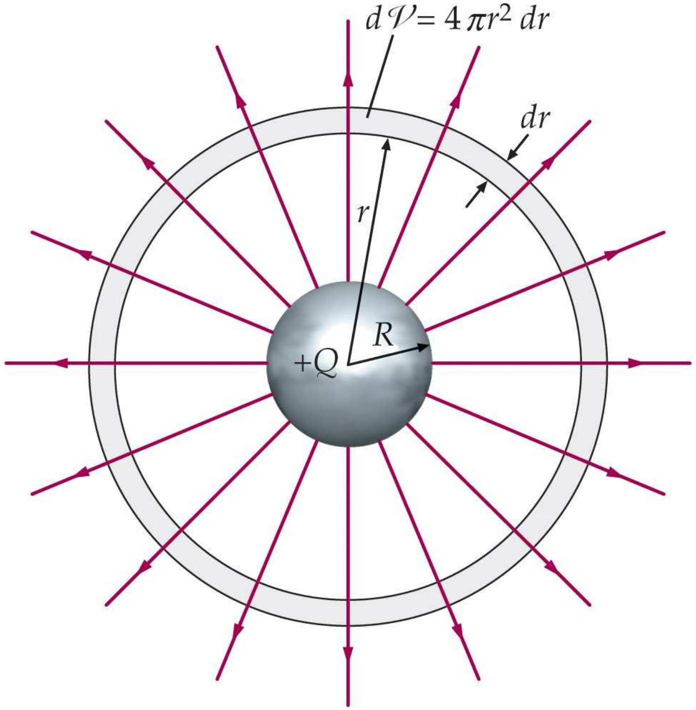
$$= \frac{Q^2}{2\epsilon_0 A}(d + \Delta z - d) = \frac{Q^2}{2\epsilon_0 A}\Delta z$$

$$F\Delta z = \Delta U \Longrightarrow F = \frac{Q^2}{2\epsilon_0 A} = \left(\frac{\sigma}{2\epsilon_0}\right) Q$$

La densitat d'energia del camp elèctric



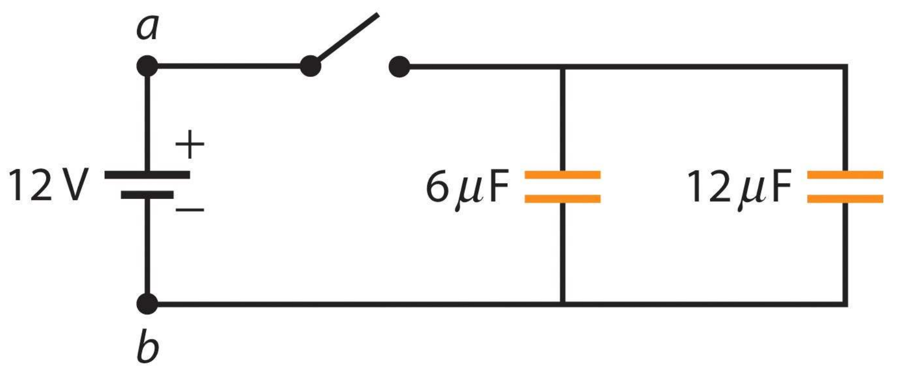
$$u_E(\vec{r}) = \frac{1}{2}\epsilon_0 \left|\vec{E}(\vec{r})\right|^2$$

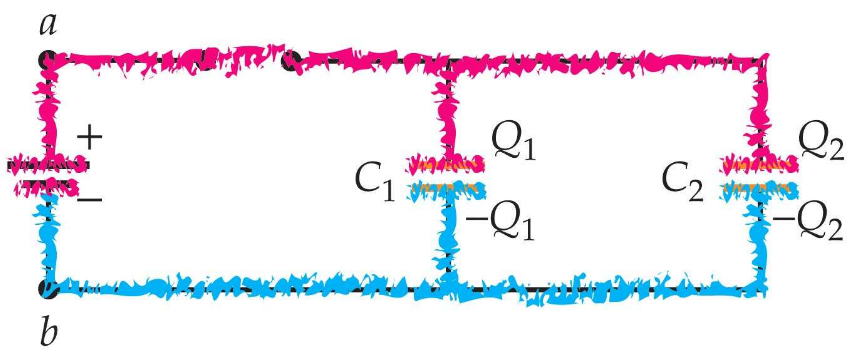


$$U = \frac{kQ^2}{2R} = \frac{1}{2}QV$$

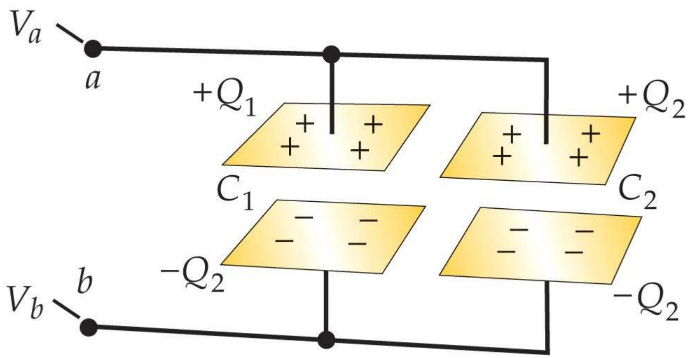
Combinacions de condensadors

circuits





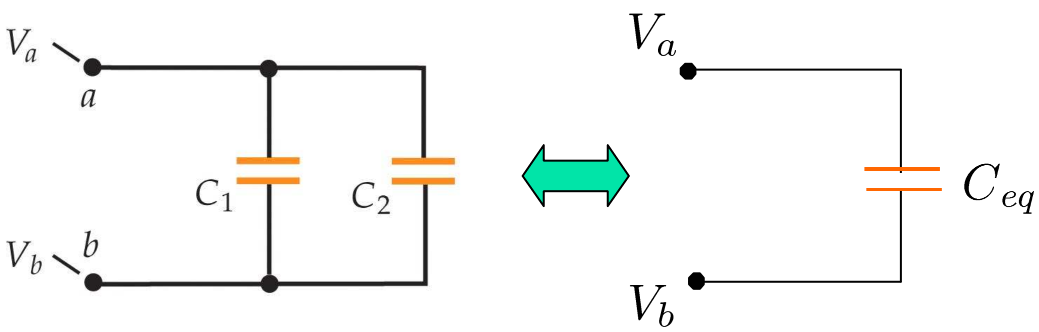
En paral.lel



$$Q_1 = C_1 V$$

$$Q_2 = C_2 V$$

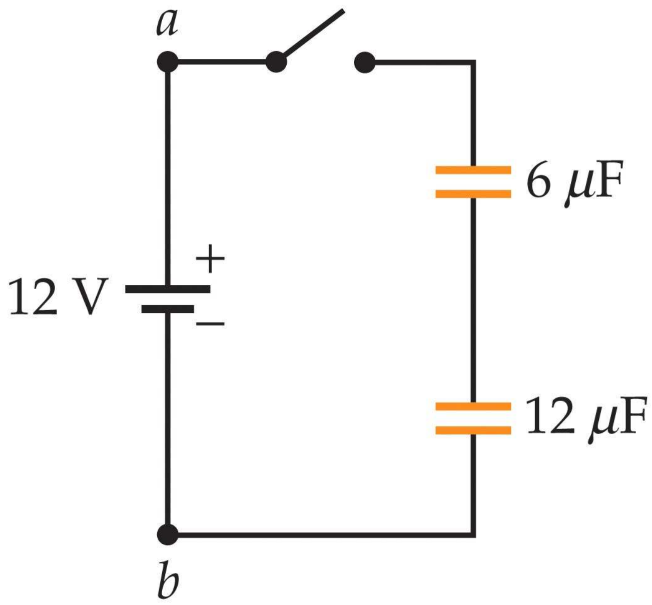
$$Q_1 + Q_2 = (C_1 + C_2)V$$

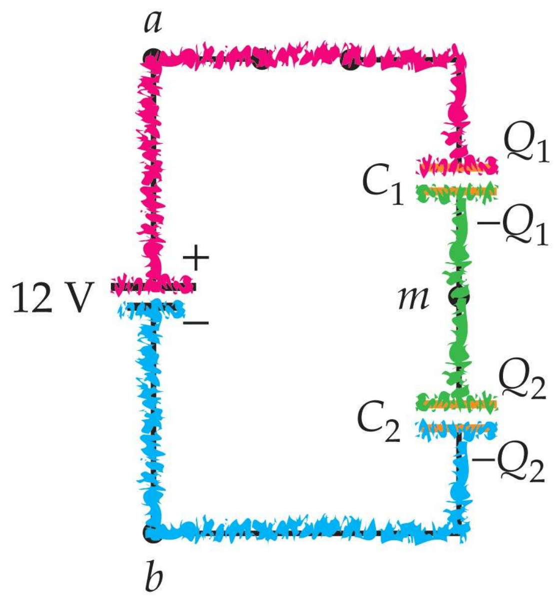


$$C_{eq} = C_1 + C_2 + C_3 + \ldots \qquad 24\text{-}17$$

EQUIVALENT CAPACITANCE FOR CAPACITORS IN PARALLEL

En sèrie





$$V_a - V_m = \frac{Q_1}{C_1}$$

$$V_m - V_b = \frac{Q_2}{C_2}$$

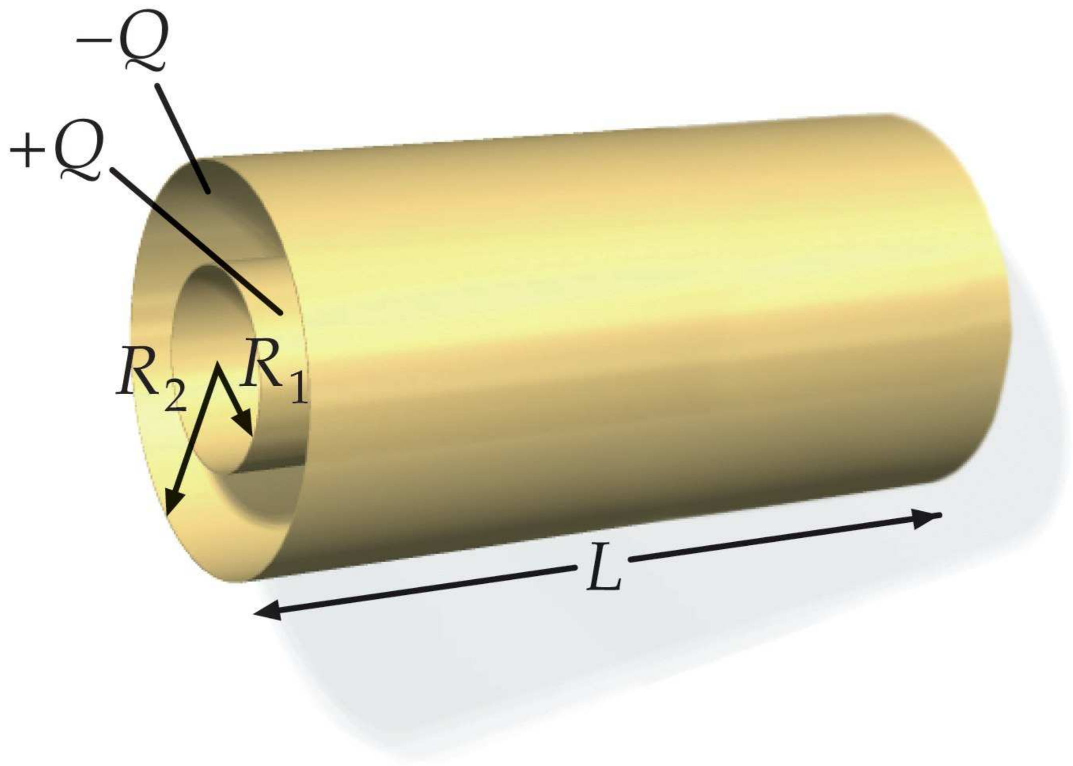
$$V_a - V_b = \frac{Q_1}{C_1} + \frac{Q_2}{C_2}$$

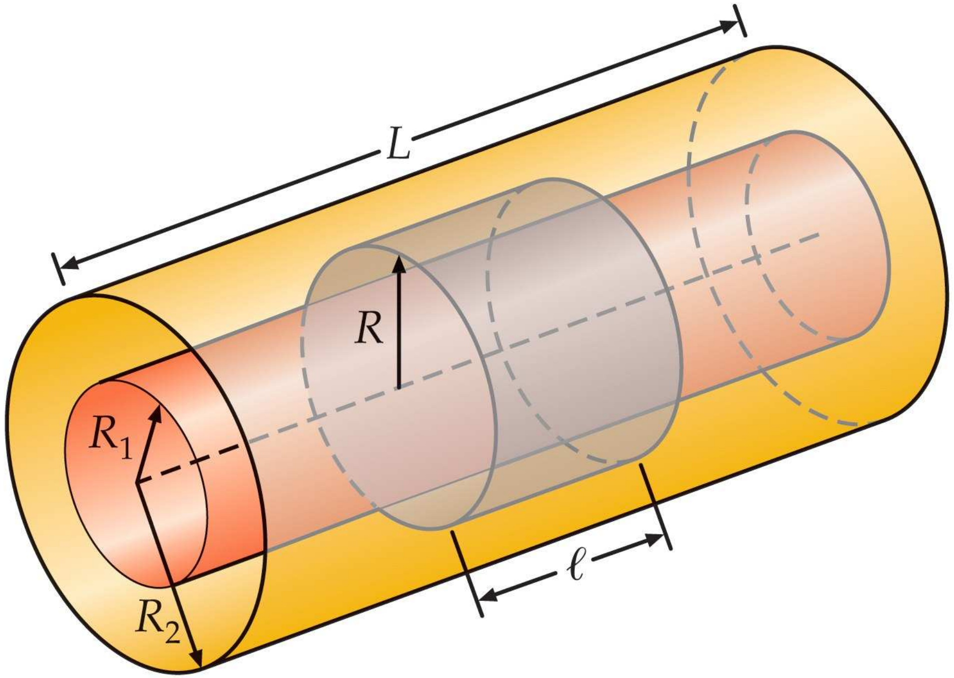
$$Q_1 = Q_2$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots \qquad 24\text{-}21$$

EQUIVALENT CAPACITANCE FOR EQUALLY CHARGED CAPACITORS IN SERIES

El condensador cilíndric





$$E\,2\pi R\ell = \frac{\sigma_1\, 2\pi R_1 \ell}{\epsilon_0}$$

$$\begin{cases} E(R) = \dfrac{\sigma_1}{\epsilon_0}\dfrac{R_1}{R} \\ \phi(R) = -\dfrac{\sigma_1 R_1}{\epsilon_0}\ln R + \text{const.} \end{cases}$$

$$V = \phi_1 - \phi_2 = \frac{\sigma_1 R_1}{\epsilon_0}\ln\frac{R_2}{R_1}$$

$$C = \frac{Q}{V} = \frac{\sigma_1\, 2\pi R_1 L}{V}$$

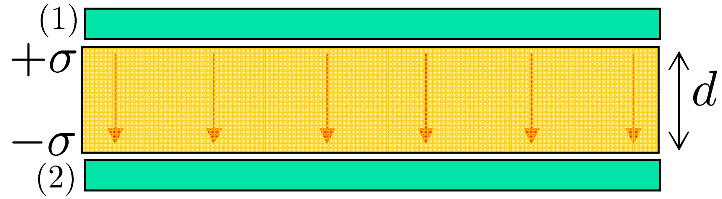
$$C = \frac{2\pi\epsilon_0 L}{\ln(R_2/R_1)} \qquad 24\text{-}11$$

Capacitance of a cylindrical capacitor

Medis dielèctrics

Aïllants o dielèctrics: les càrregues no es poden moure lliurement al seu interior.

Condensador amb dielèctric



$$C = \frac{\epsilon_0 A}{d} \quad \Rightarrow \quad C = \kappa \frac{\epsilon_0 A}{d}$$

constant dielèctrica

$$C = \frac{\epsilon A}{d}$$

Permitivitat del medi

$$\epsilon = \kappa \epsilon_0$$

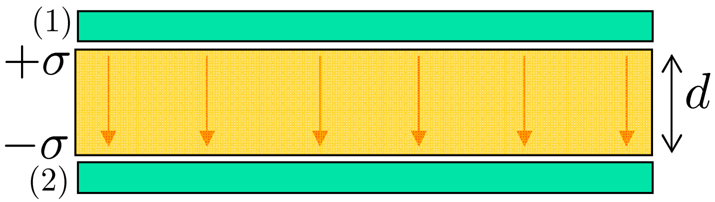
La constant dielèctrica κ és un nombre adimensional
és una caracteristica del medi
al buit és $\kappa = 1$
en general és $\kappa > 1$

TABLE 24-1

Dielectric Constants and Dielectric Strengths of Various Materials

Material	Dielectric Constant κ	Dielectric Strength, kV/mm
Air	1.00059	3
Bakelite	4.9	24
Glass (Pyrex)	5.6	14
Mica	5.4	10–100
Neoprene	6.9	12
Paper	3.7	16
Paraffin	2.1–2.5	10
Plexiglas	3.4	40
Polystyrene	2.55	24
Porcelain	7	5.7
Transformer oil	2.24	12

Per què augmenta la capacitat?



$$C = \frac{Q}{V}$$

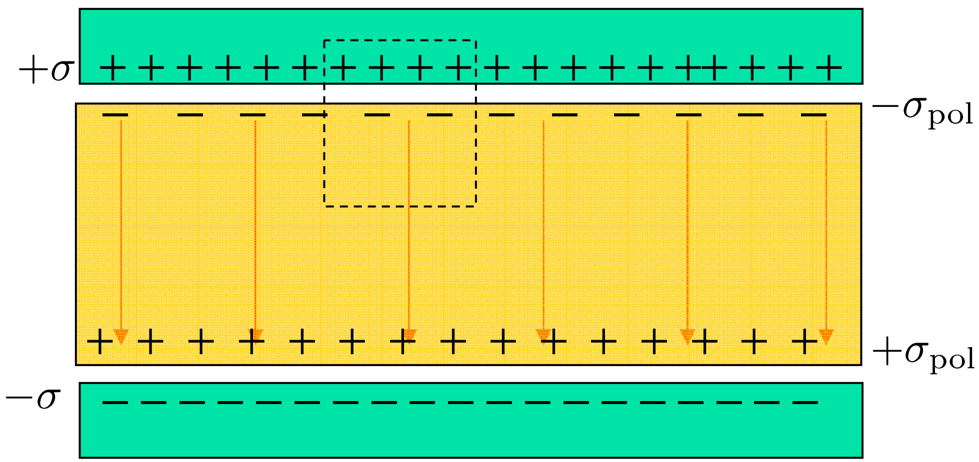
Disminueix V

$$V = -\int_2^1 \vec{E} \cdot d\vec{\ell} = Ed$$

Disminueix E

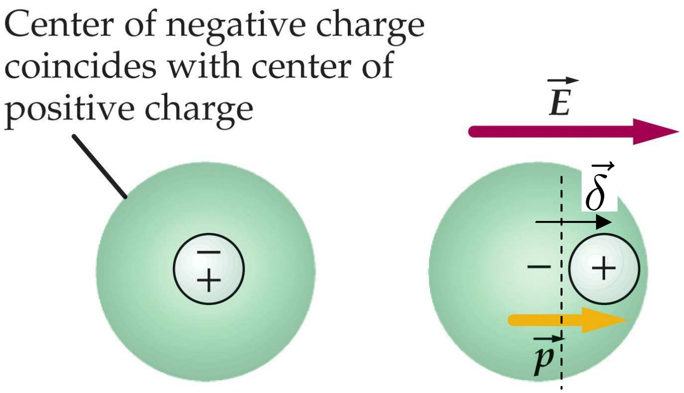
$$E = \frac{\sigma}{\kappa \epsilon_0}$$

Les càrregues als conductors són les mateixes
Apareixen càrregues a les superficies dels dielèctrics s'anomenen de polarització



$$E = \frac{\sigma - \sigma_{\mathrm{pol}}}{\epsilon_0} < \frac{\sigma}{\epsilon_0}$$

Origen de les càrregues de polarització

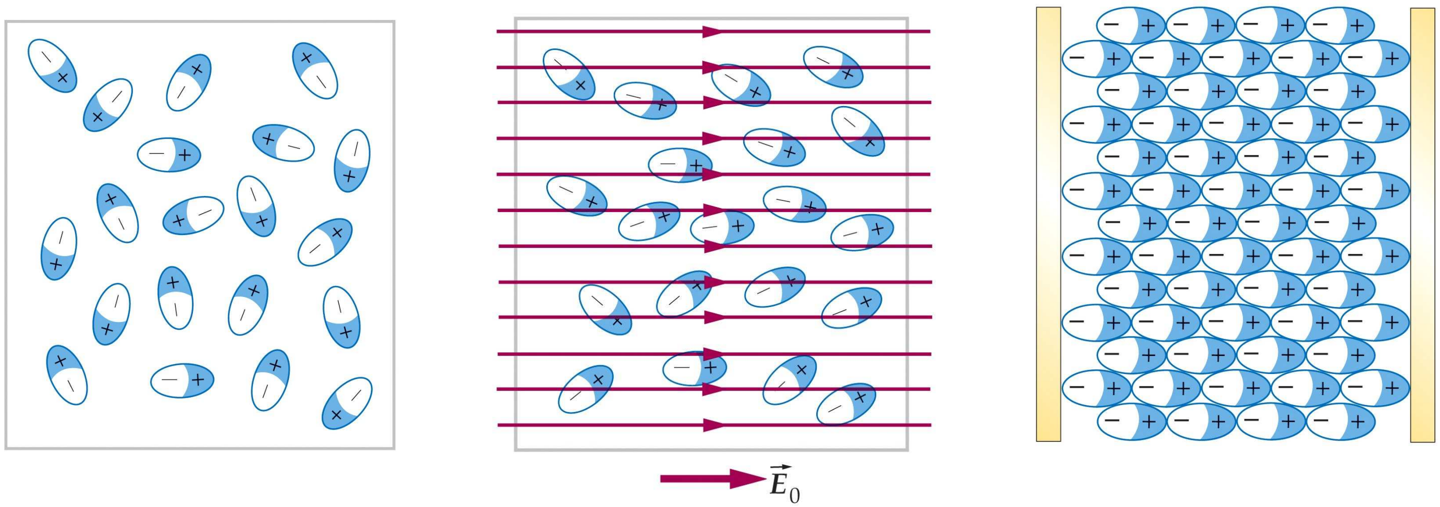


$$\vec{p} = q\vec{\delta}$$

àtoms per unitat de volum

Camp de polarització del dielèctric:
Moment dipolar *per unitat de volum*

$$\vec{P} = Nq\vec{\delta}$$

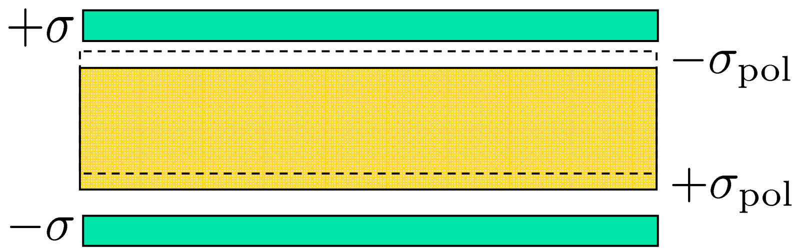


La susceptibilitat dielèctrica χ

$$\vec{P} = \epsilon_0 \chi \vec{E}$$

$$\kappa = 1 + \chi$$

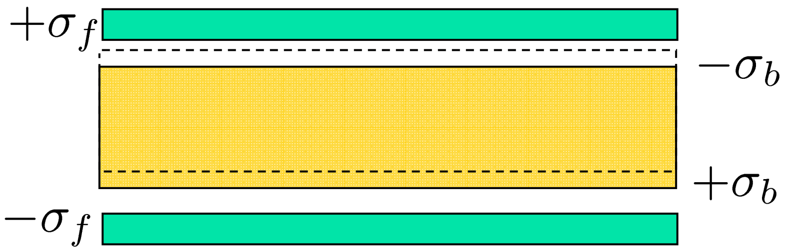
Relació entre $\vec{P}$ i les càrregues induïdes



$$A\sigma_{\rm pol} = (A\delta)Nq$$

$$\sigma_{\rm pol} = Nq\delta = |\vec{P}| = P$$

Notació: $\begin{cases} \sigma \equiv \sigma_f & \text{als conductors (free)} \\ \sigma_{\rm pol} \equiv \sigma_b & \text{als dielèctrics (bound)} \end{cases}$



$$E = \frac{\sigma_f - \sigma_b}{\epsilon_0}$$

$$\sigma_b = P = \chi \epsilon_0 E$$

$$E = \frac{\sigma_f - P}{\epsilon_0}$$

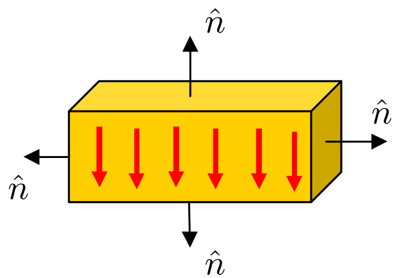
$$E = \frac{\sigma_f}{\epsilon_0} - \chi E$$

$$E = \frac{\sigma_f}{(1+\chi)\epsilon_0}$$

Recordem que $E = \dfrac{\sigma}{\kappa \epsilon_0}$ $\Rightarrow$ $\kappa = 1 + \chi$

En general, a cada superfície del dielèctric

$\vec{P} = \epsilon_0 \chi \vec{E}$



$$\sigma_b = \vec{P} \cdot \hat{n}$$

La llei de Gauss amb dielèctrics: el desplaçament elèctric

càrrega total (lliure més lligada)

$$\oint_S \vec{E} \cdot d\vec{s} = \frac{Q_{\rm int}}{\epsilon_0}$$

Volem una llei nomes amb la càrrega lliure (no de polarització)

El camp de desplaçament elèctric

$$\vec{D}(\vec{r}) = \epsilon_0 \vec{E}(\vec{r}) + \vec{P}(\vec{r})$$

$$\vec{D}(\vec{r}) = \epsilon_0 \vec{E}(\vec{r}) + \chi\epsilon_0 \vec{E}(\vec{r}) = \epsilon_0(1+\chi)\vec{E}(\vec{r})$$

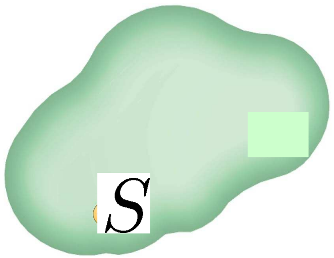
$$\boxed{\vec{D}(\vec{r}) = \epsilon_0 \kappa \, \vec{E}(\vec{r})}$$

Es sol definir la *permitivitat del medi* $\epsilon = \epsilon_0 \kappa = \epsilon_0(1+\chi)$

La llei del desplaçament

$$\int_S \vec{D} \cdot \vec{ds} = Q_f^{\text{int}}$$

càrrega lliure a l'interior de S



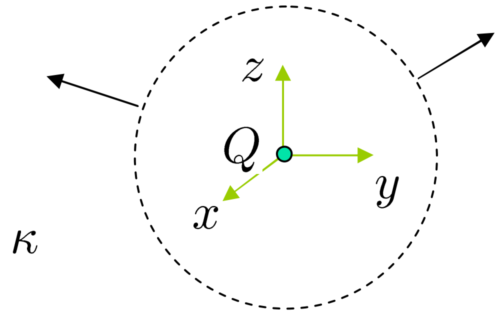
Procediment general:

$$\vec{D}(\vec{r}) \Rightarrow \vec{E}(\vec{r}) = \frac{\vec{D}(\vec{r})}{\epsilon_0 \kappa} \Rightarrow \vec{P}(\vec{r}) = \chi \epsilon_0 \, \vec{E}(\vec{r}) \Rightarrow \sigma_b = \vec{P} \cdot \vec{n}$$

No necessitam coneixer les càrregues de polarització
Les podem determinar al final
El desplaçament és independent dels dielèctrics
Els dielèctrics redueixen els camps (apantallament parcial)

Exemples

Càrrega puntual en un medi dielèctric infinit

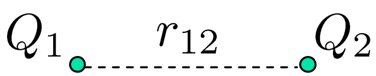


$$\vec{D} = D(r)\hat{u}_r$$

$$D\,4\pi r^2 = Q \Rightarrow D(r) = \frac{Q}{4\pi r^2}$$

$$E(r) = \frac{Q}{\underbrace{4\pi\epsilon_0\kappa}_{\epsilon} r^2}$$

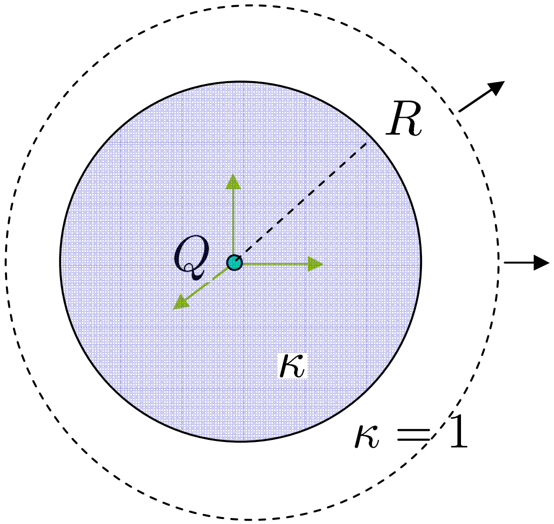
Força entre dues càrregues



$$F = \frac{Q_1 Q_2}{4\pi\epsilon r_{12}^2}$$

Reducció del camp i de la força Coulomb $\epsilon_0 \rightarrow \epsilon$

Càrrega puntual al centre d'una esfera dielèctrica



$$D\, 4\pi r^2 = Q \Rightarrow D(r) = \frac{Q}{4\pi r^2}$$

$$E(r) = \frac{Q}{4\pi\epsilon_0 \kappa r^2} \qquad P(r) = \chi\epsilon_0\, E(r)$$

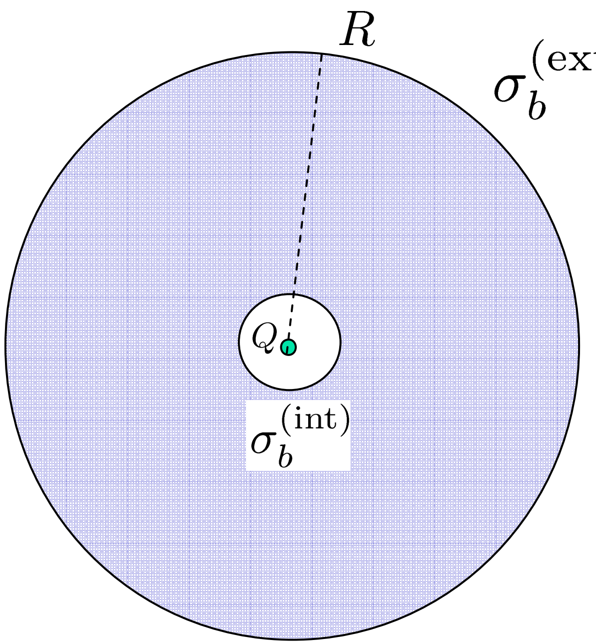
	$r < R$	$r > R$
desplaçament	$\frac{Q}{4\pi r^2}$	$\frac{Q}{4\pi r^2}$
camp	$\frac{Q}{4\pi\epsilon_0 \kappa r^2}$	$\frac{Q}{4\pi\epsilon_0 r^2}$
polarització	$\frac{\kappa-1}{\kappa}\frac{Q}{4\pi r^2}$	0

Densitats induïdes

$$\text{a } r = R, \qquad \sigma_b = \vec{P}(R)\cdot\hat{u}_r = \frac{\kappa-1}{\kappa}\frac{Q}{4\pi R^2}$$

$$\text{a } r = \delta, \qquad \sigma_b = -\vec{P}(\delta)\cdot\hat{u}_r = -\frac{\kappa-1}{\kappa}\frac{Q}{4\pi \delta^2}$$

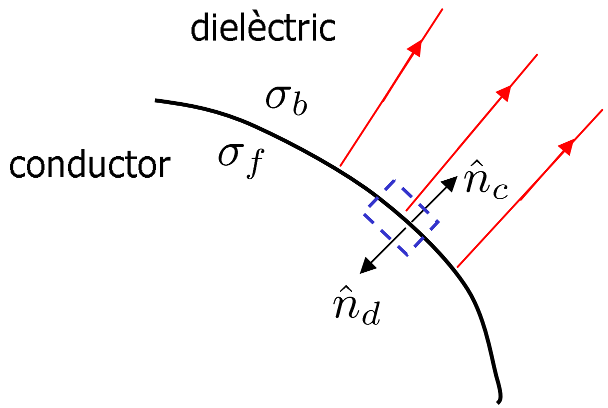
Neutralitat del dielèctric



$$\sigma_b^{(\mathrm{ext})} = \frac{\kappa - 1}{\kappa} \frac{Q}{4\pi R^2}$$

$$\sigma_b^{(\mathrm{int})} = -\frac{\kappa - 1}{\kappa} \frac{Q}{4\pi \delta^2}$$

La interfase conductor-dielèctric



$$\oint_S \vec{D} \cdot \vec{ds} = Q_f^{\text{int}}$$

$$DA = \sigma_f A \quad \Rightarrow \quad \vec{D} = \sigma_f \hat{n}_c$$

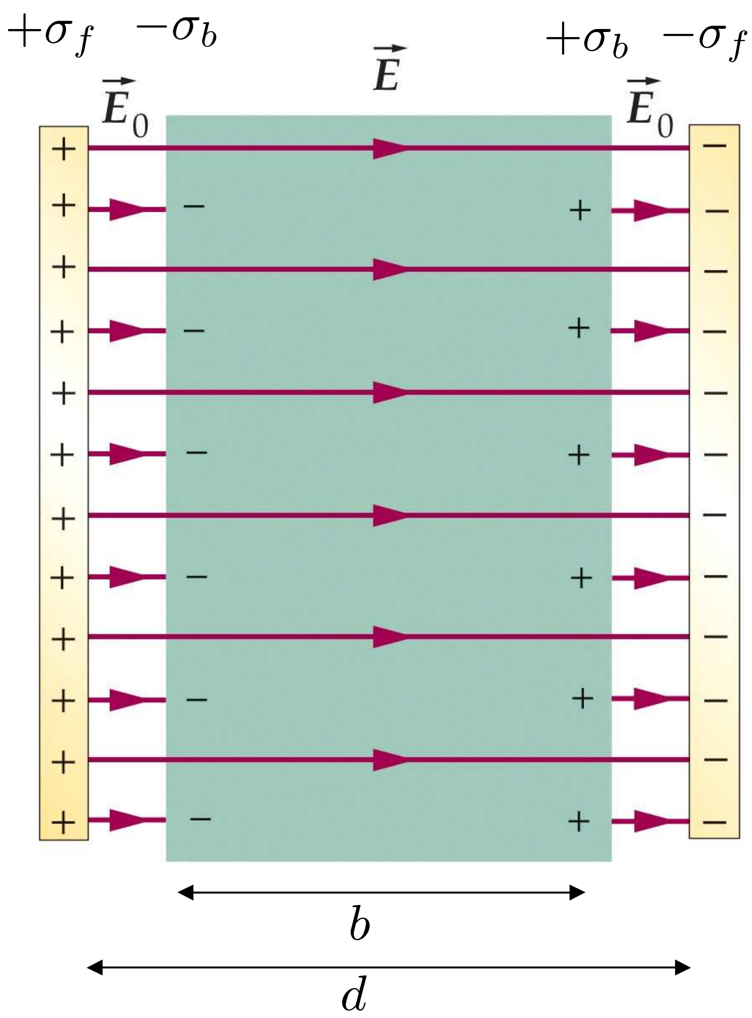
$$\vec{E} = \frac{\sigma_f}{\epsilon_0 \kappa} \hat{n}_c$$

$$\vec{P} = \frac{\kappa - 1}{\kappa} \sigma_f \hat{n}_c$$

$$\sigma_b = \vec{P} \cdot \hat{n}_d = \frac{\kappa - 1}{\kappa} \sigma_f \hat{n}_c \cdot \hat{n}_d = -\frac{\kappa - 1}{\kappa} \sigma_f$$

$$\boxed{\sigma_b = -\frac{\kappa - 1}{\kappa} \sigma_f}$$

Condensador amb dielèctric parcial



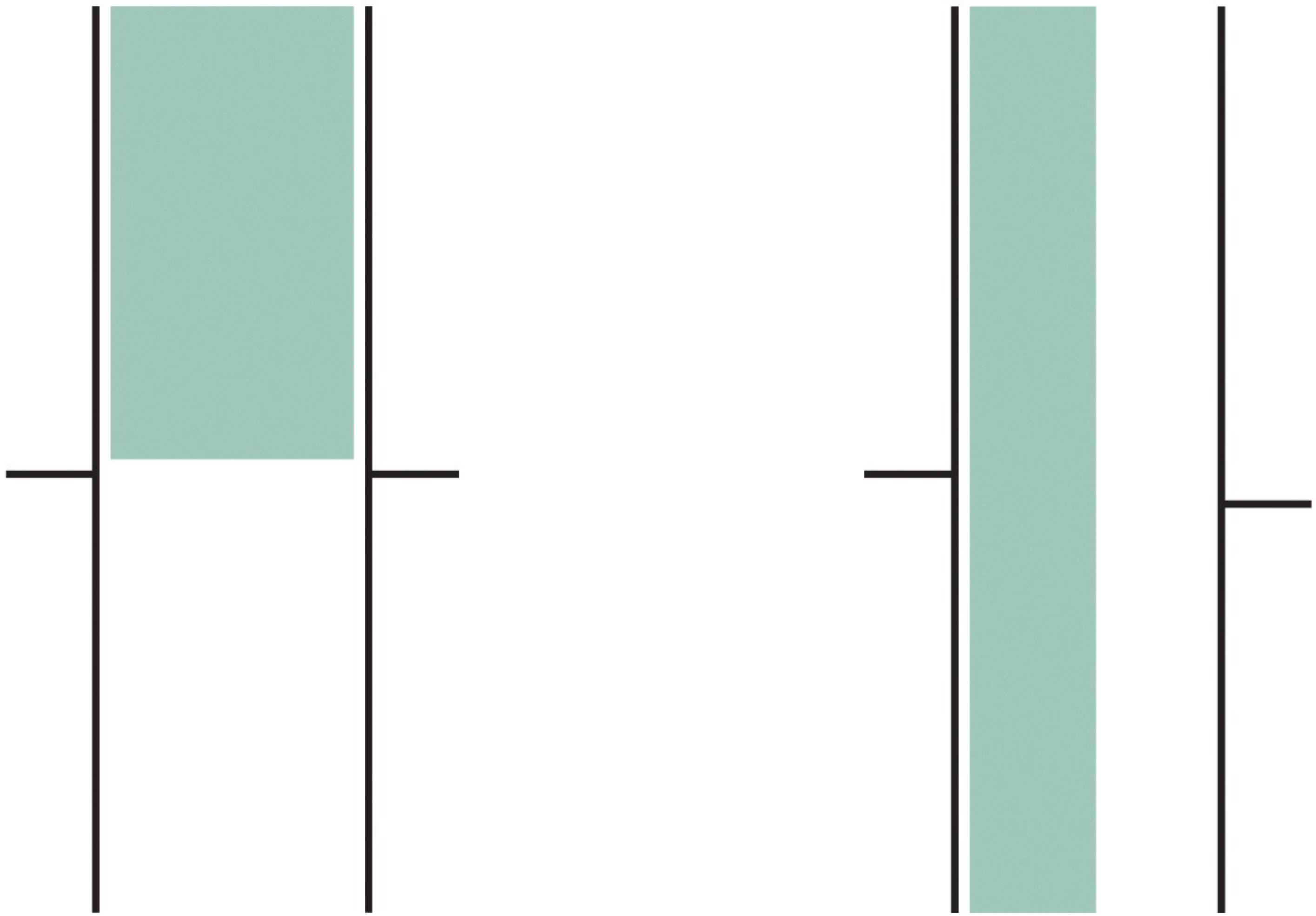
$$D = \sigma_f \qquad \begin{cases} E_0 = \dfrac{\sigma_f}{\epsilon_0} \\ E = \dfrac{\sigma_f}{\epsilon_0 \kappa} \end{cases}$$

$$\begin{aligned} V &= \frac{\sigma_f}{\epsilon_0}\ell_1 + \frac{\sigma_f}{\epsilon_0 \kappa}\ell_2 + \frac{\sigma_f}{\epsilon_0}\ell_3 \\ &= \frac{\sigma_f}{\epsilon_0}(d-b) + \frac{\sigma_f}{\epsilon_0 \kappa} b \end{aligned}$$

$$\begin{aligned} C &= \frac{Q_f}{V} = \frac{\sigma_f A}{V} \\ &= \frac{\epsilon_0 A}{d} \frac{1}{1 - \frac{b}{d}\frac{\kappa - 1}{\kappa}} \end{aligned}$$

Problems

24.10



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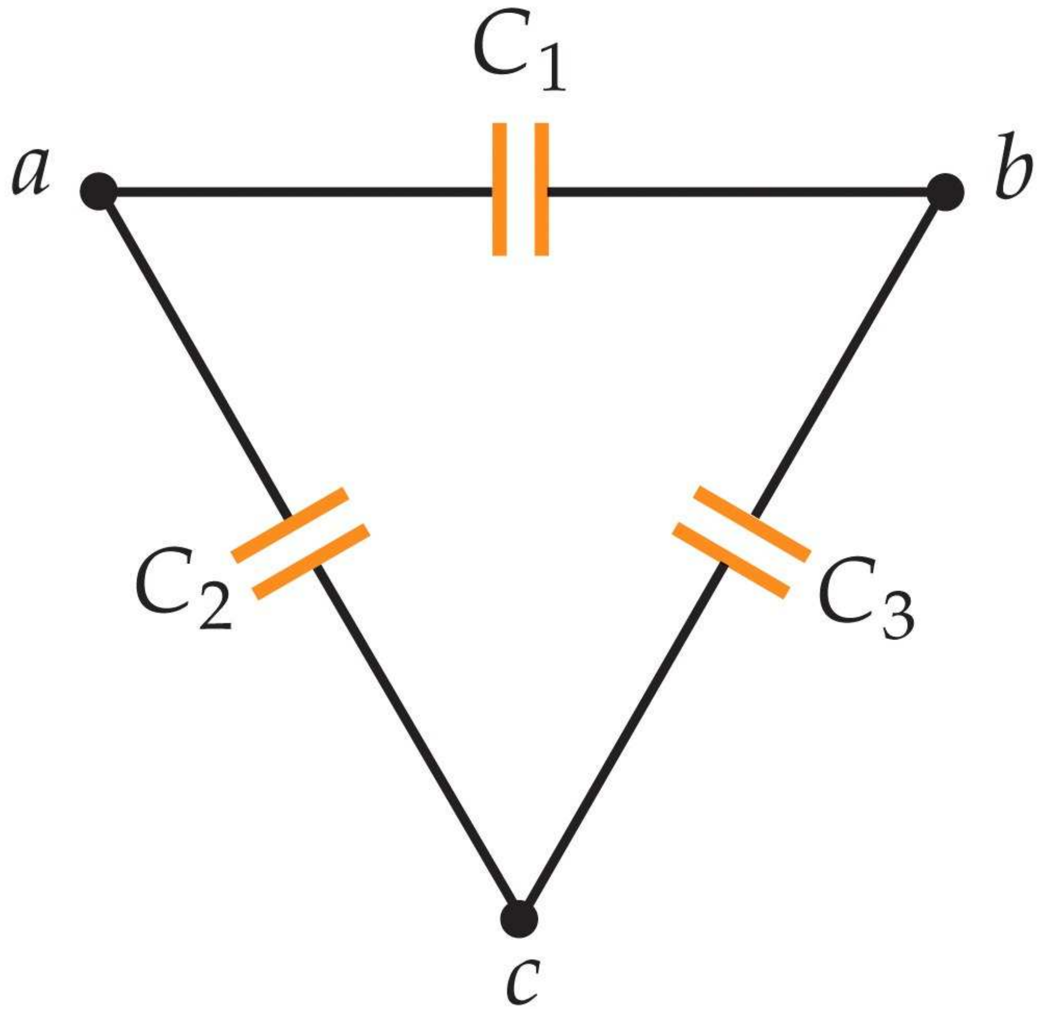

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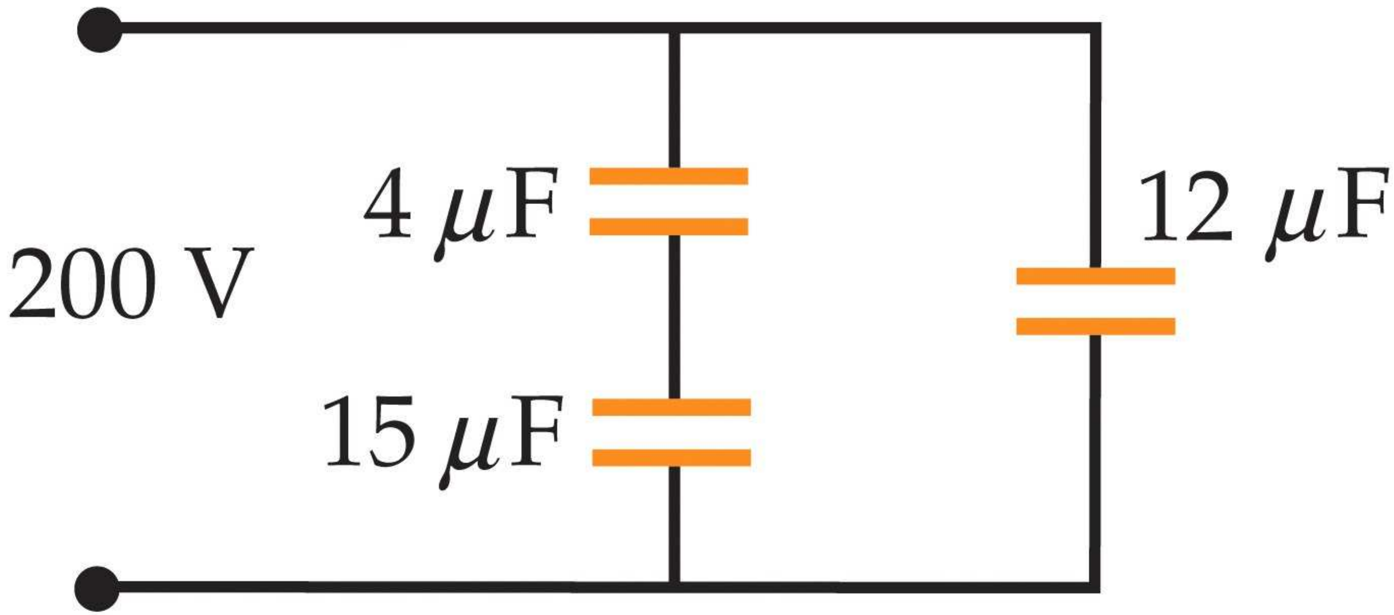

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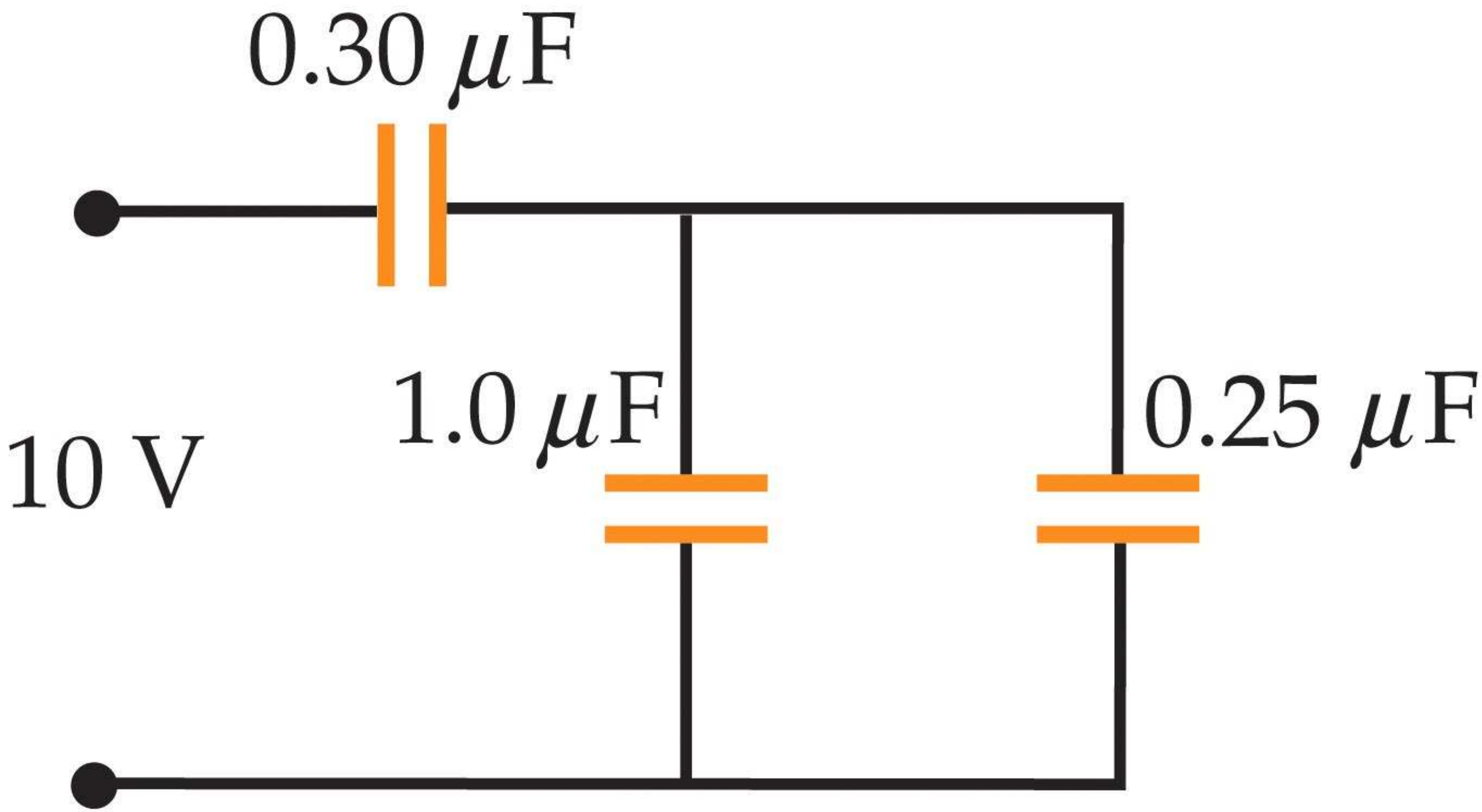

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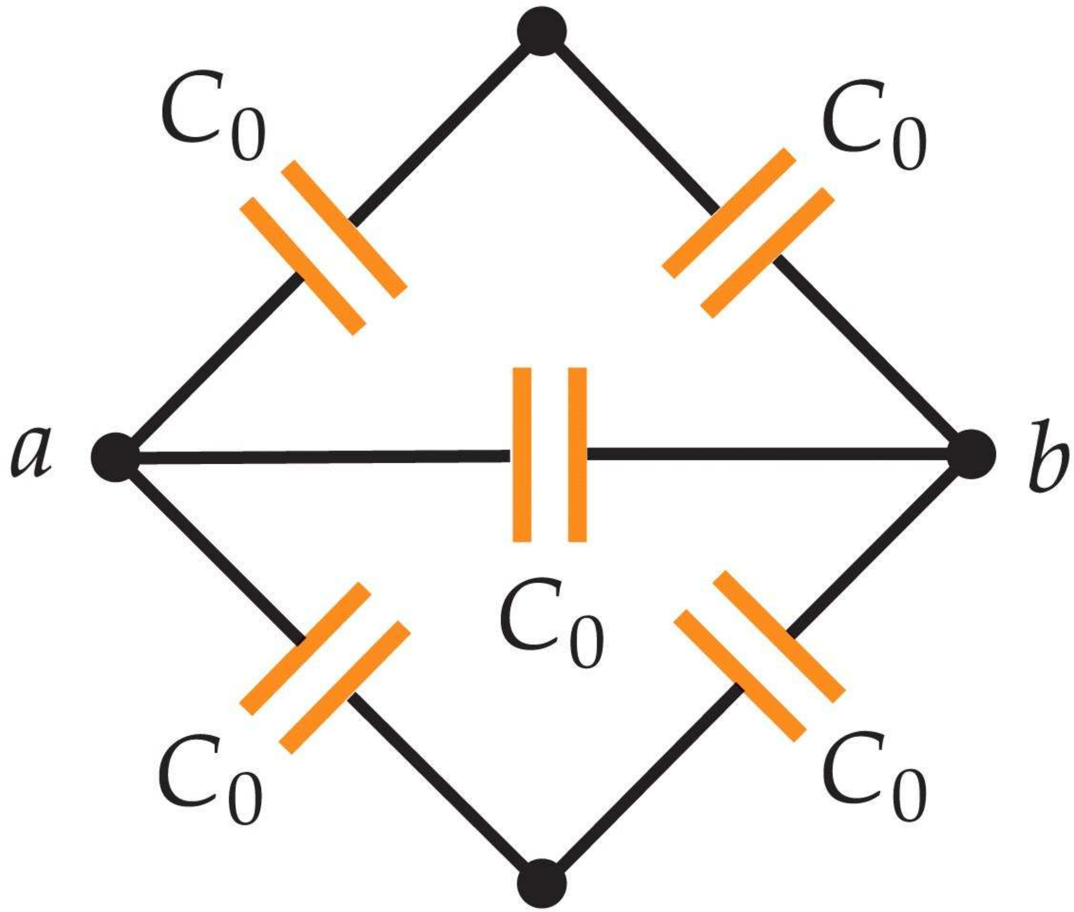

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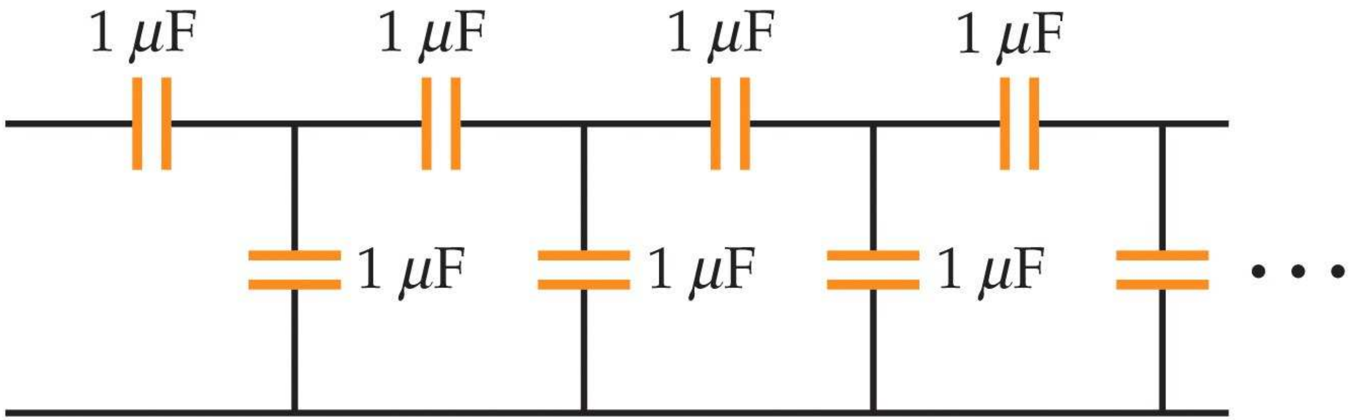

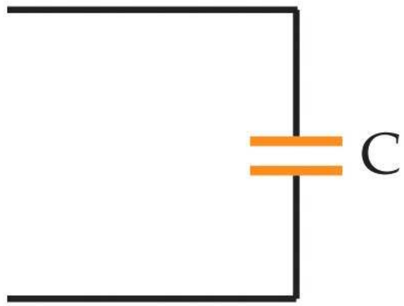

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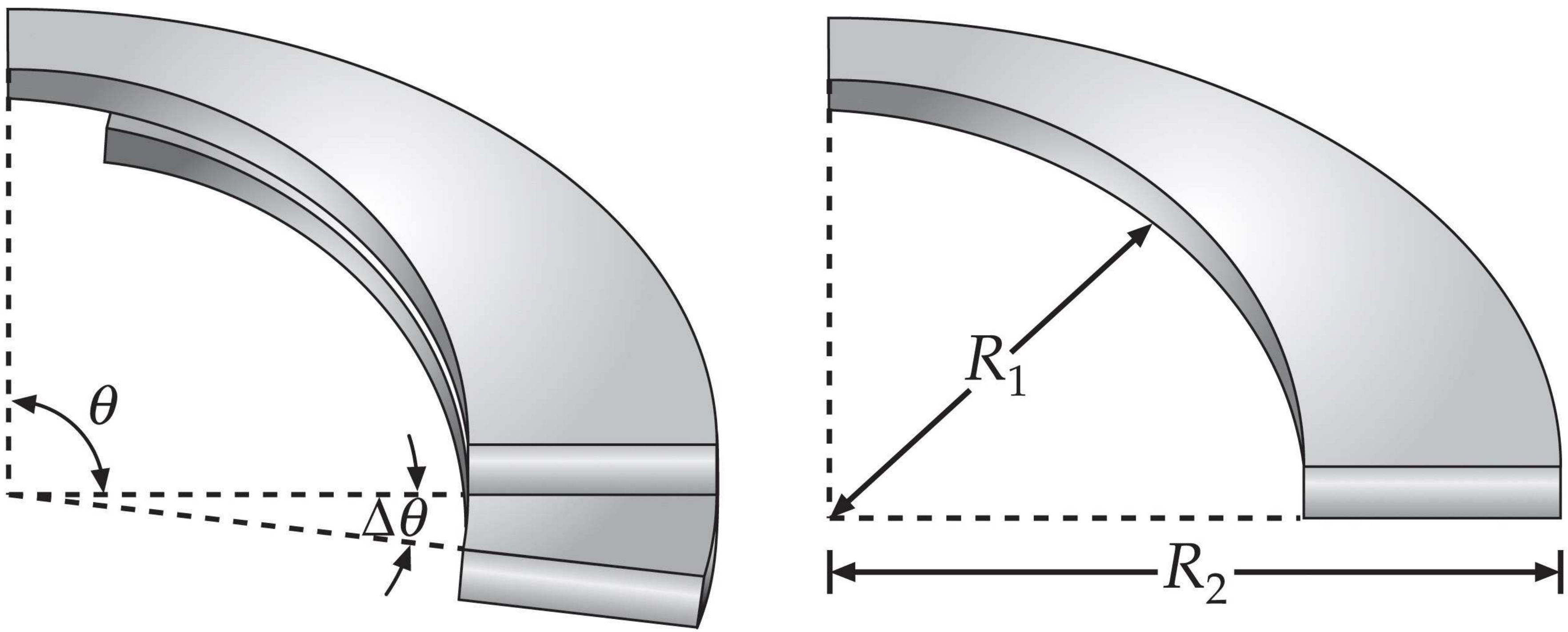

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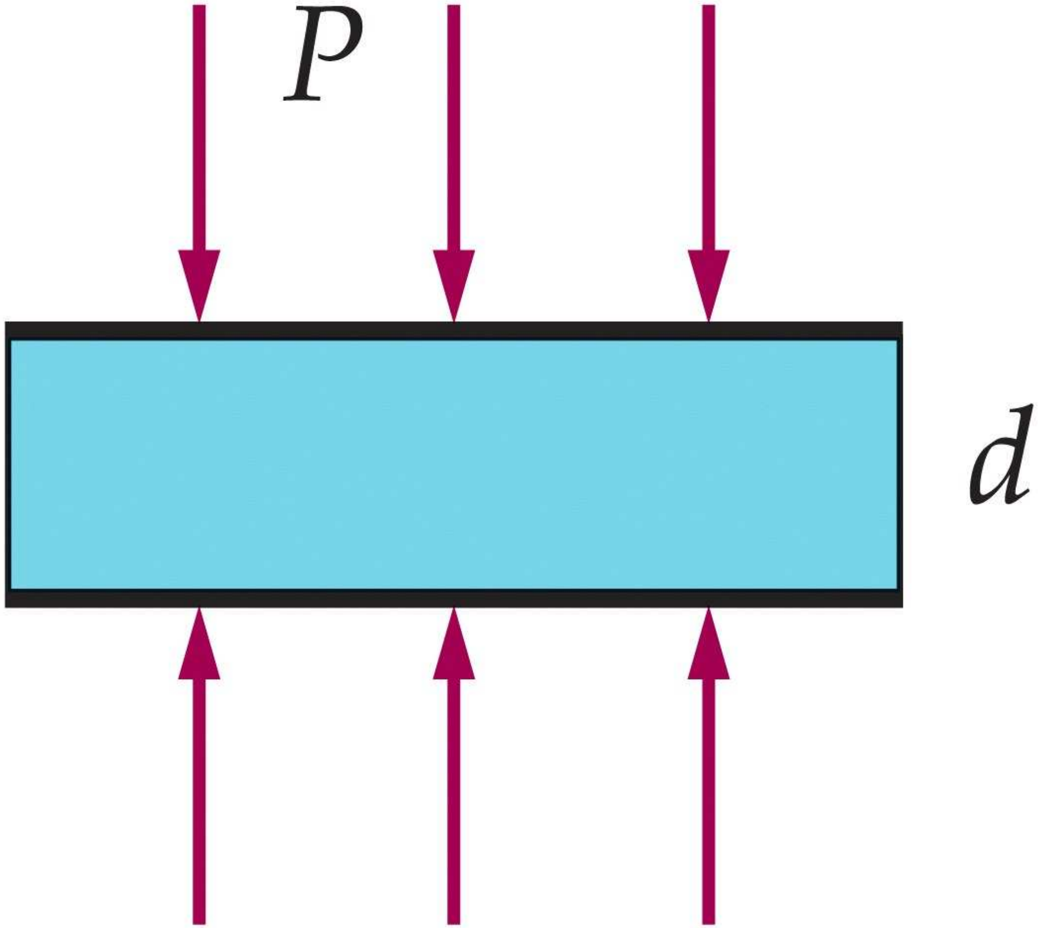

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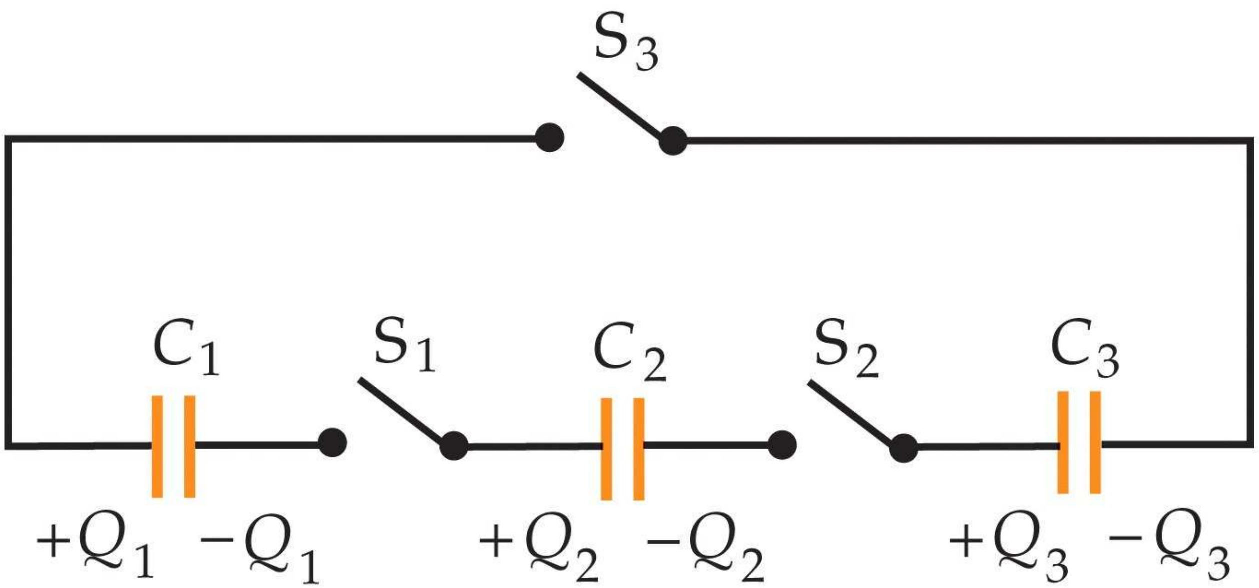

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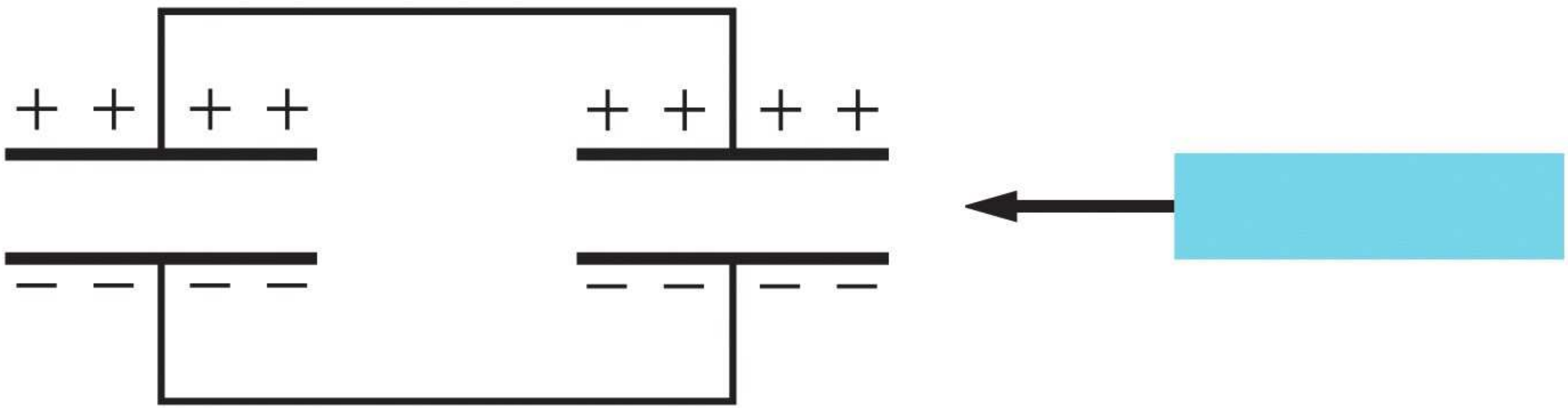

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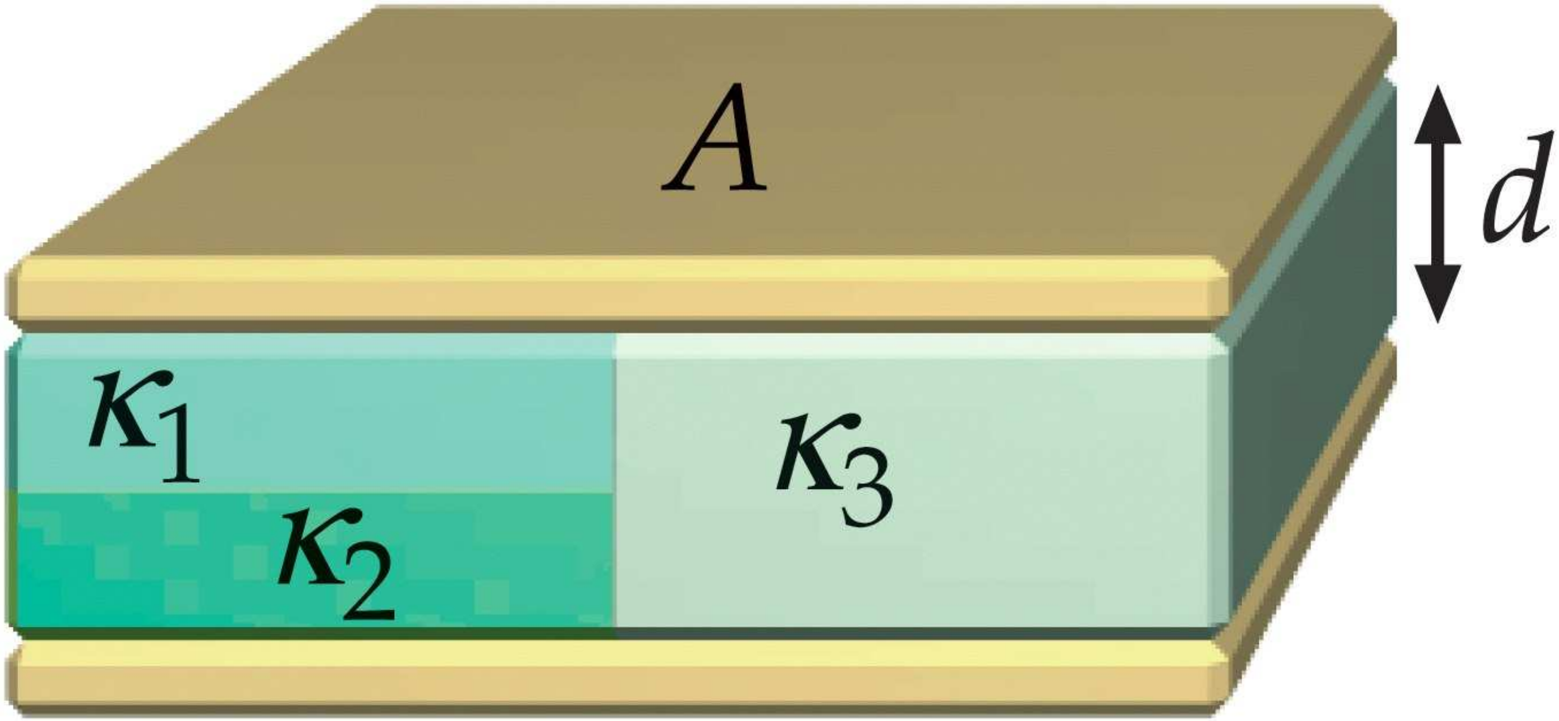

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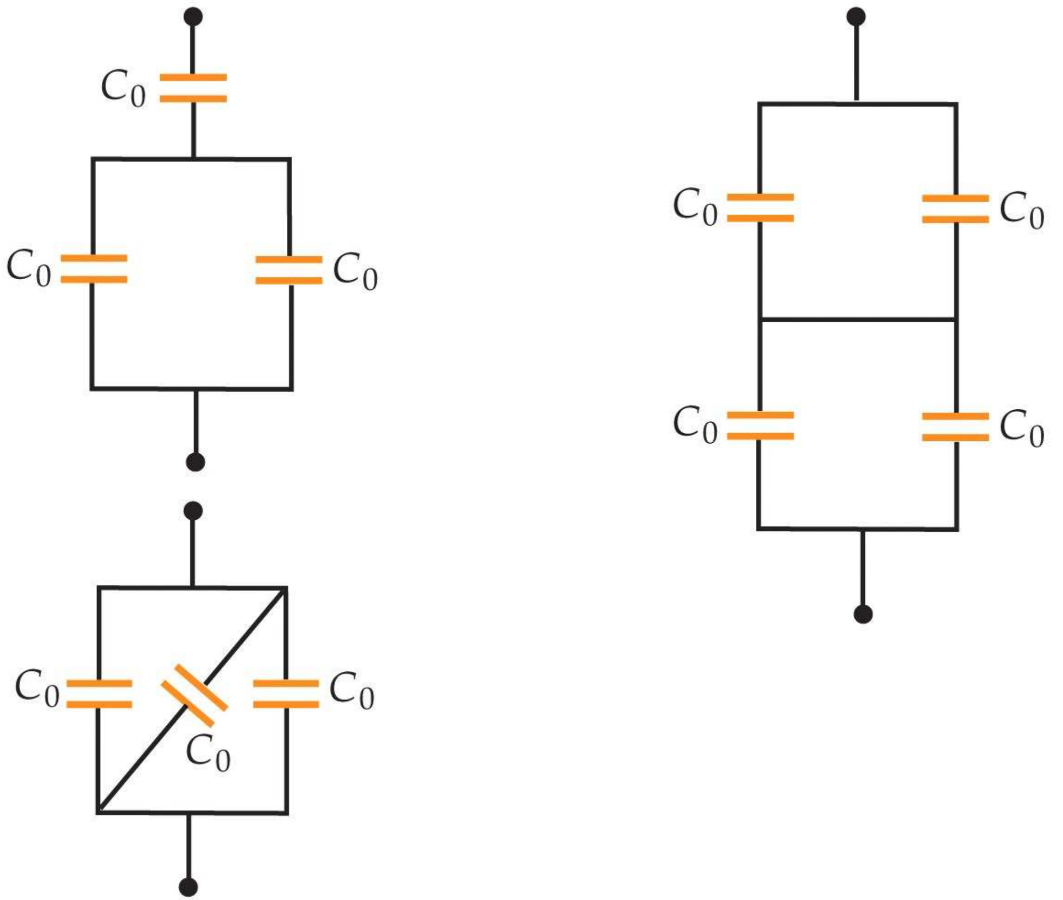

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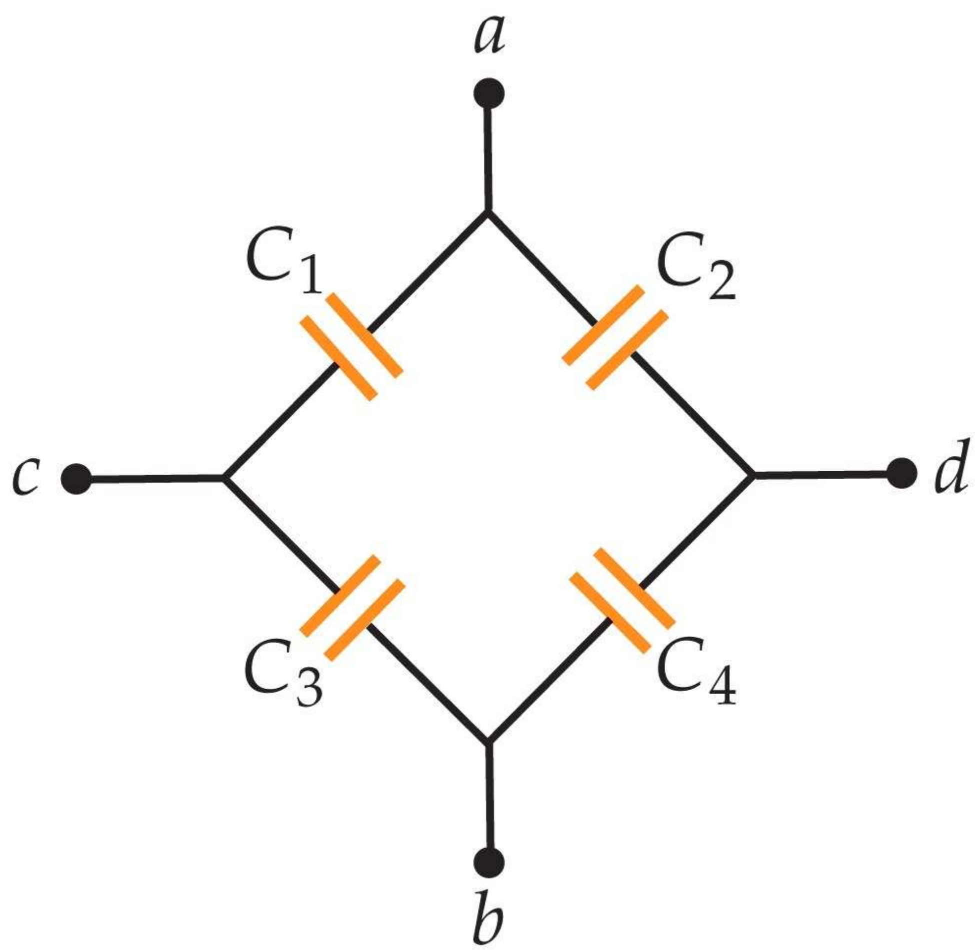

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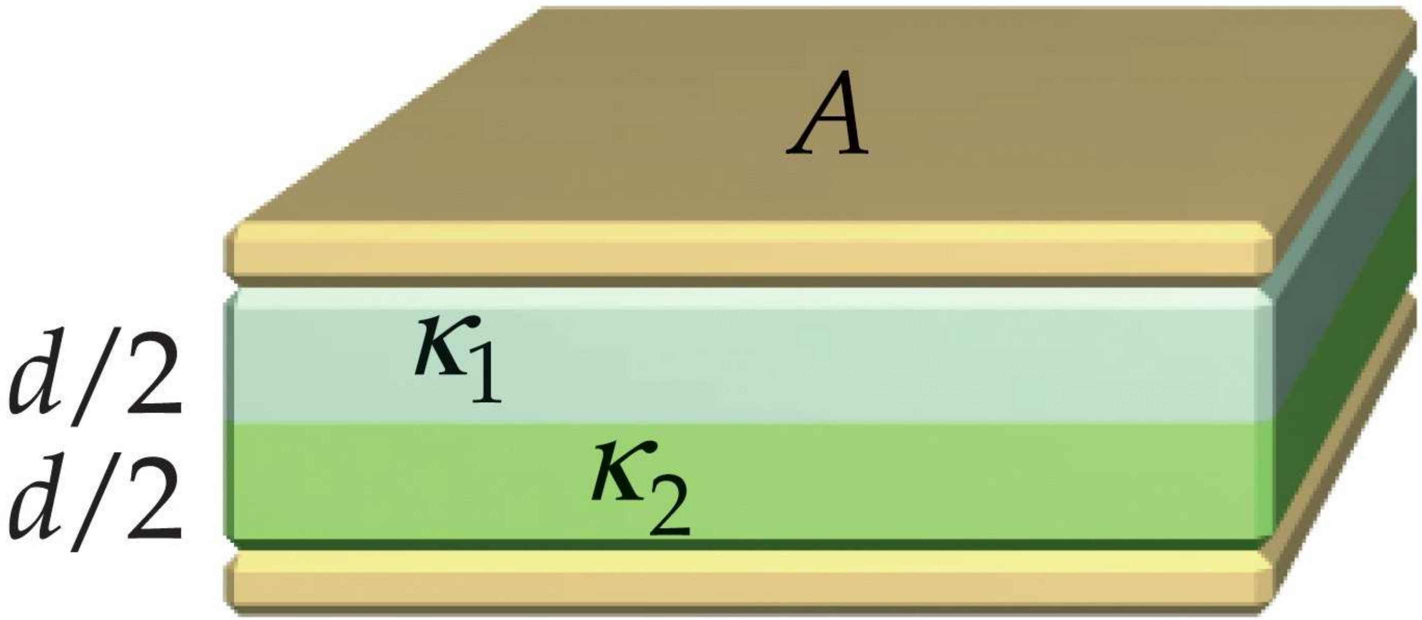

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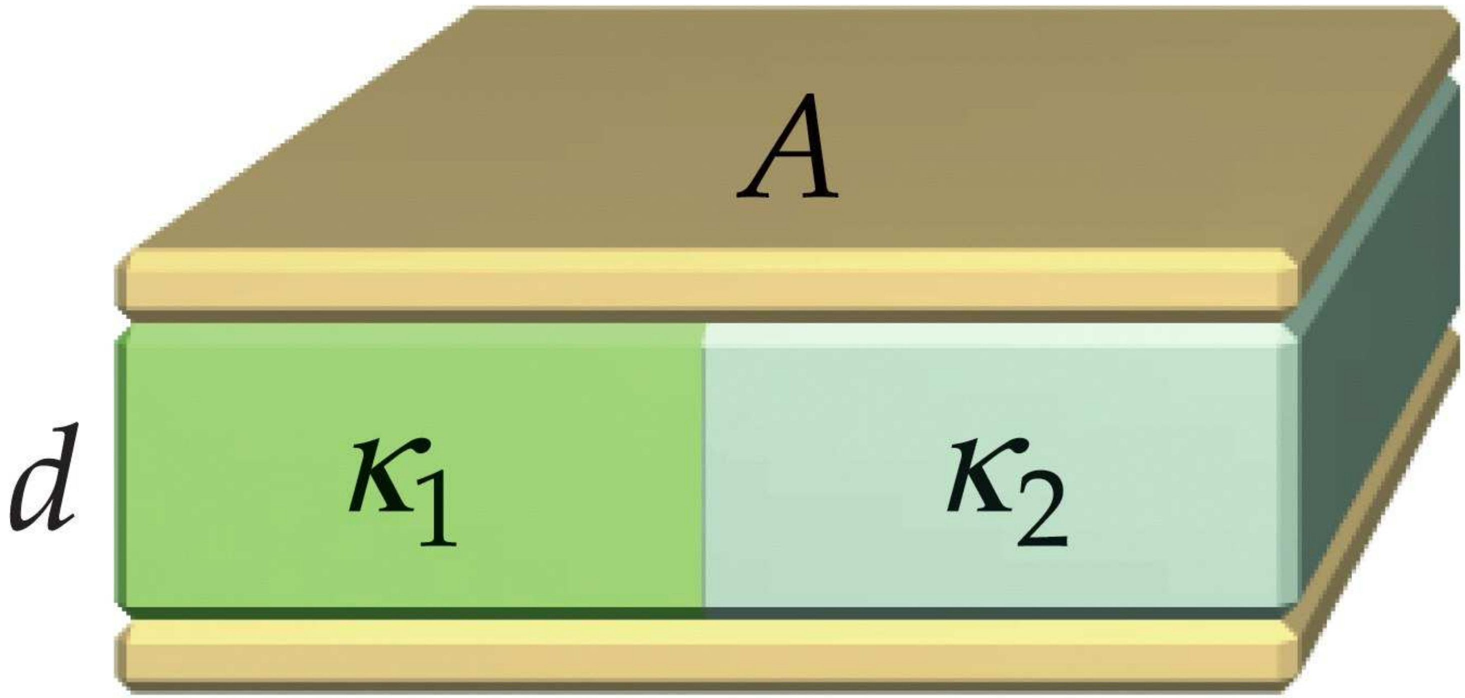

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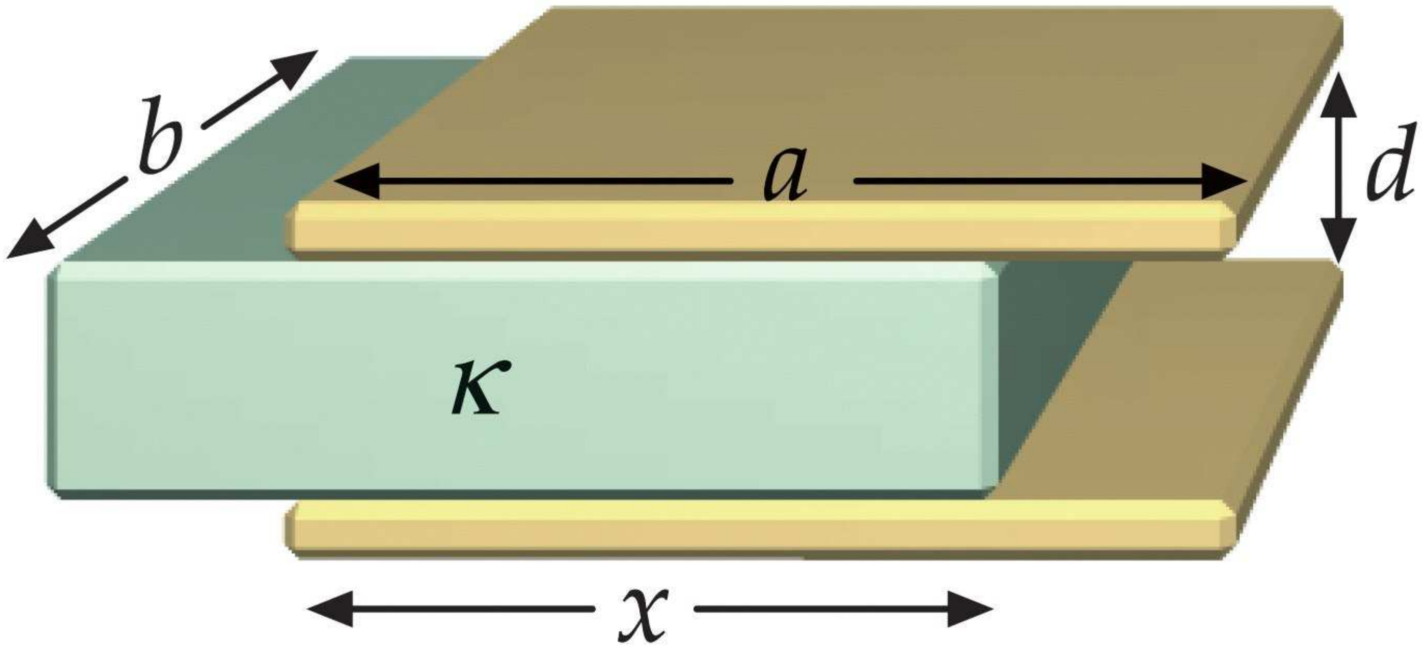

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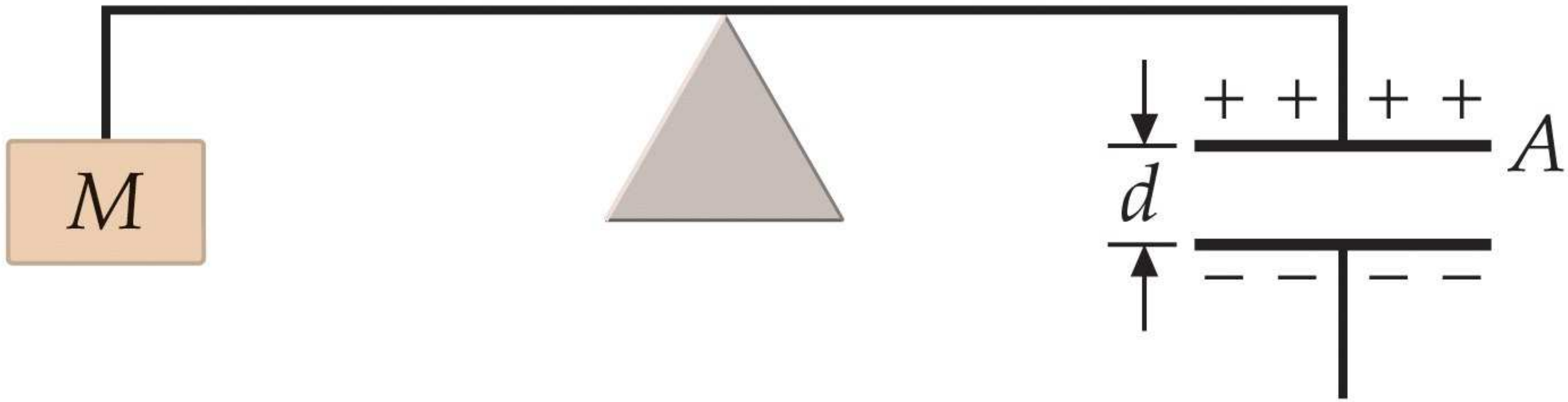

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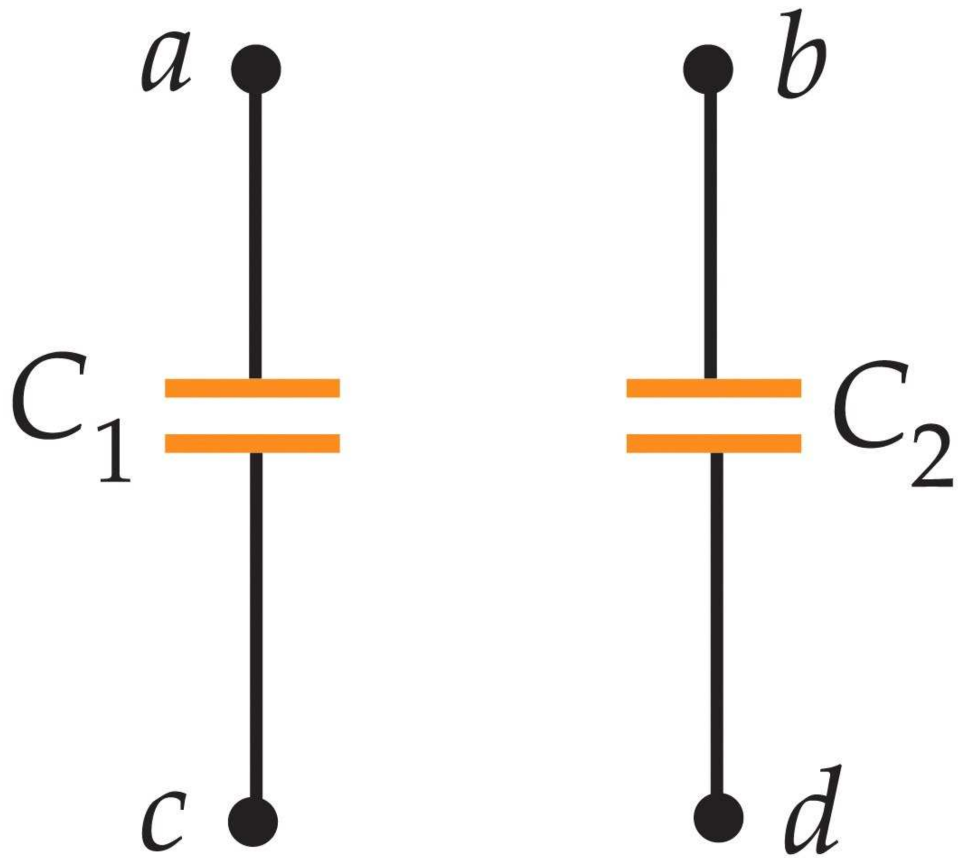

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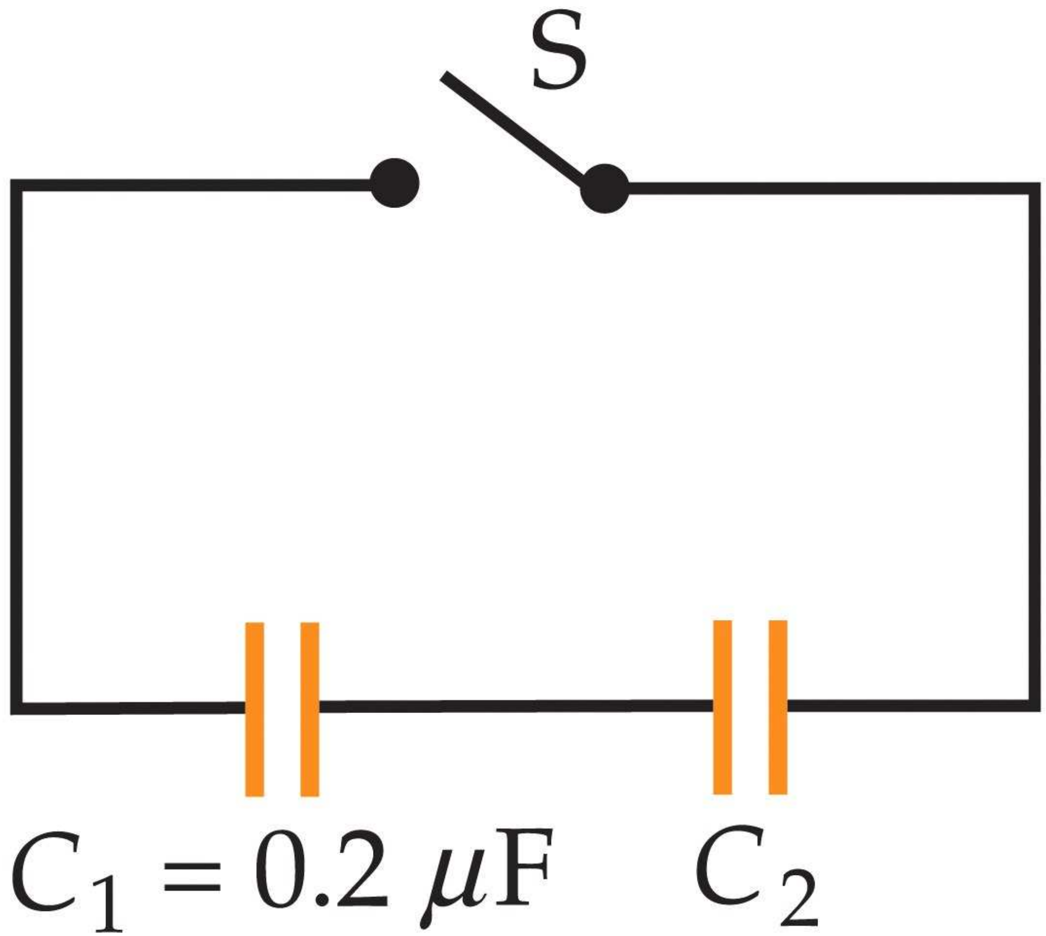

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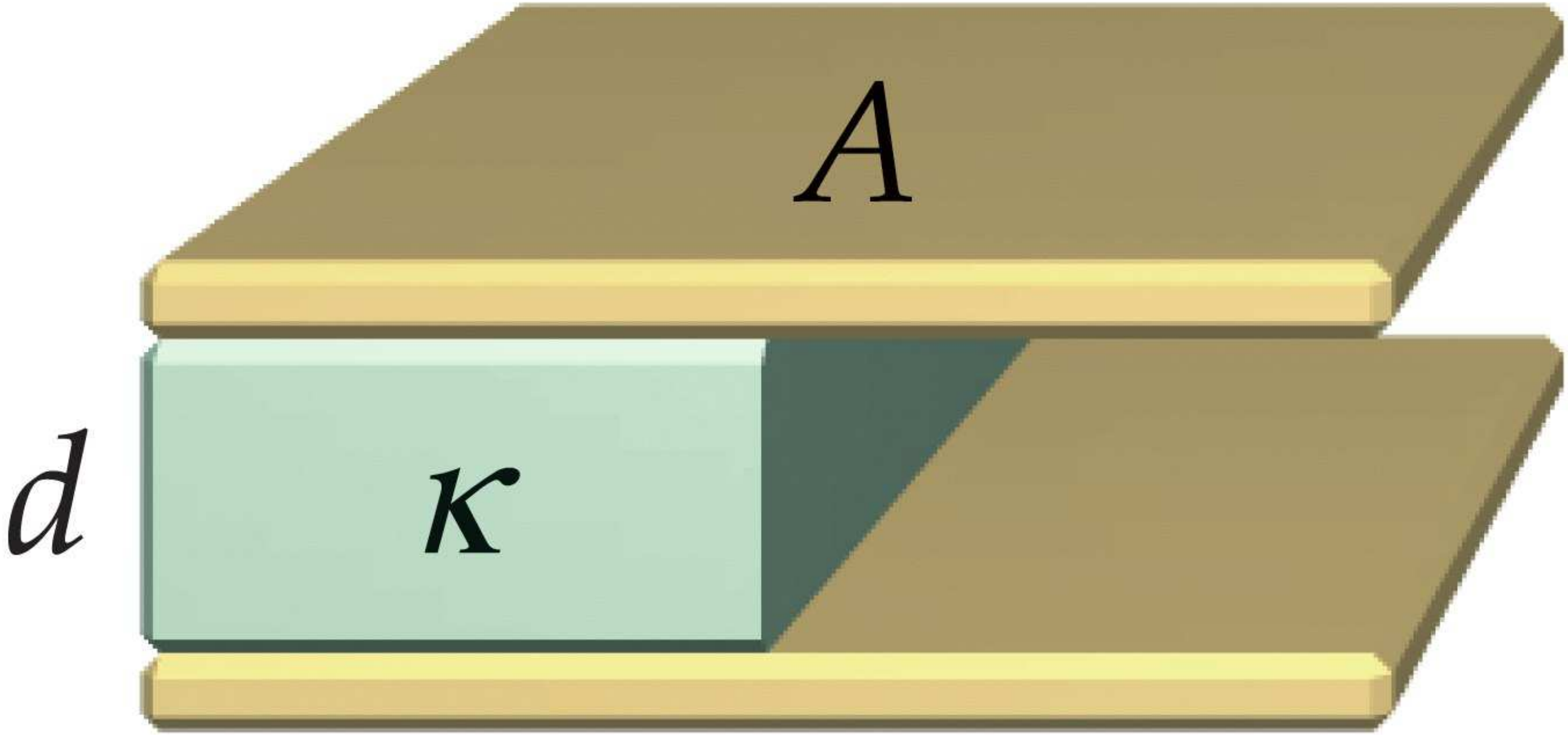

24.110