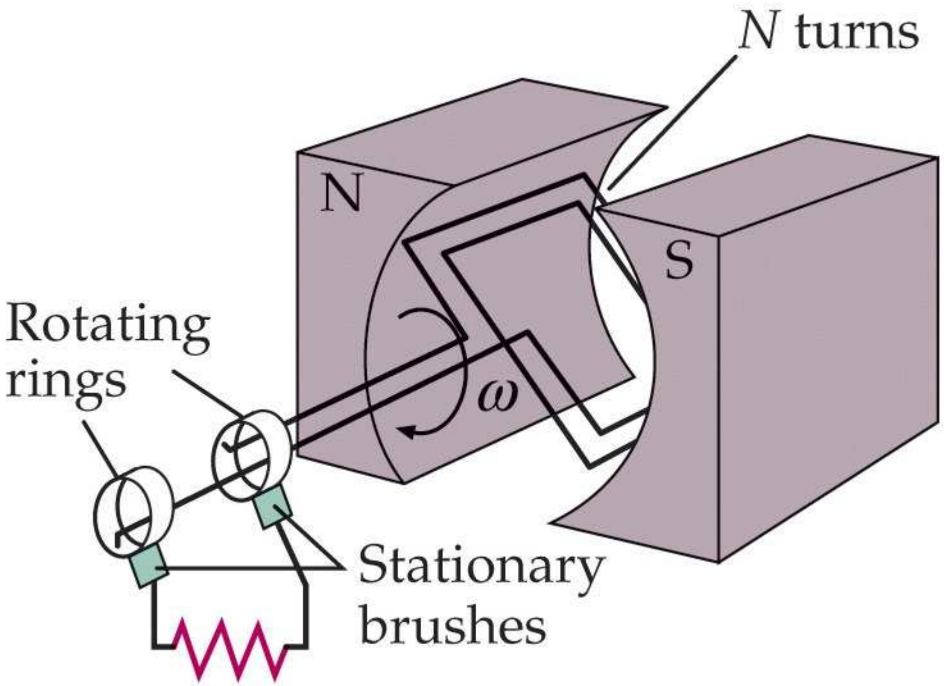
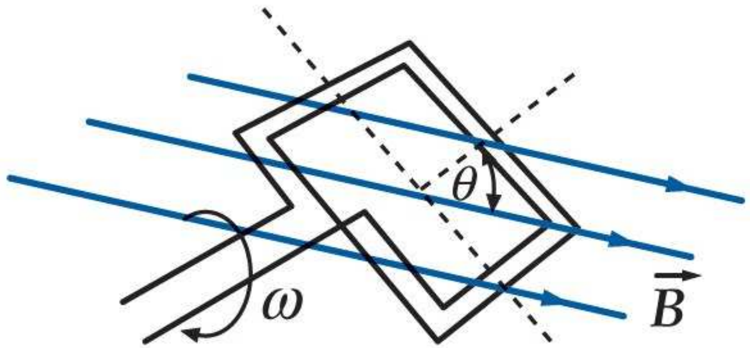
Circuits AC

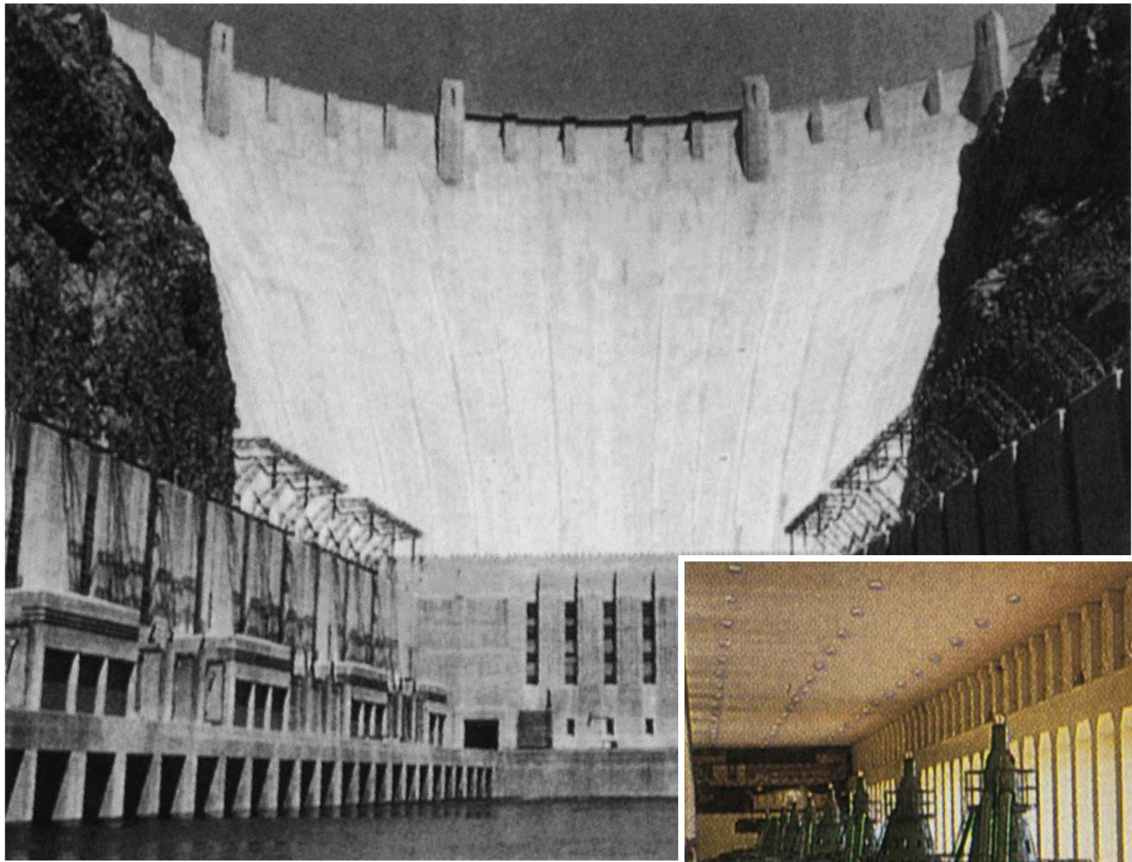
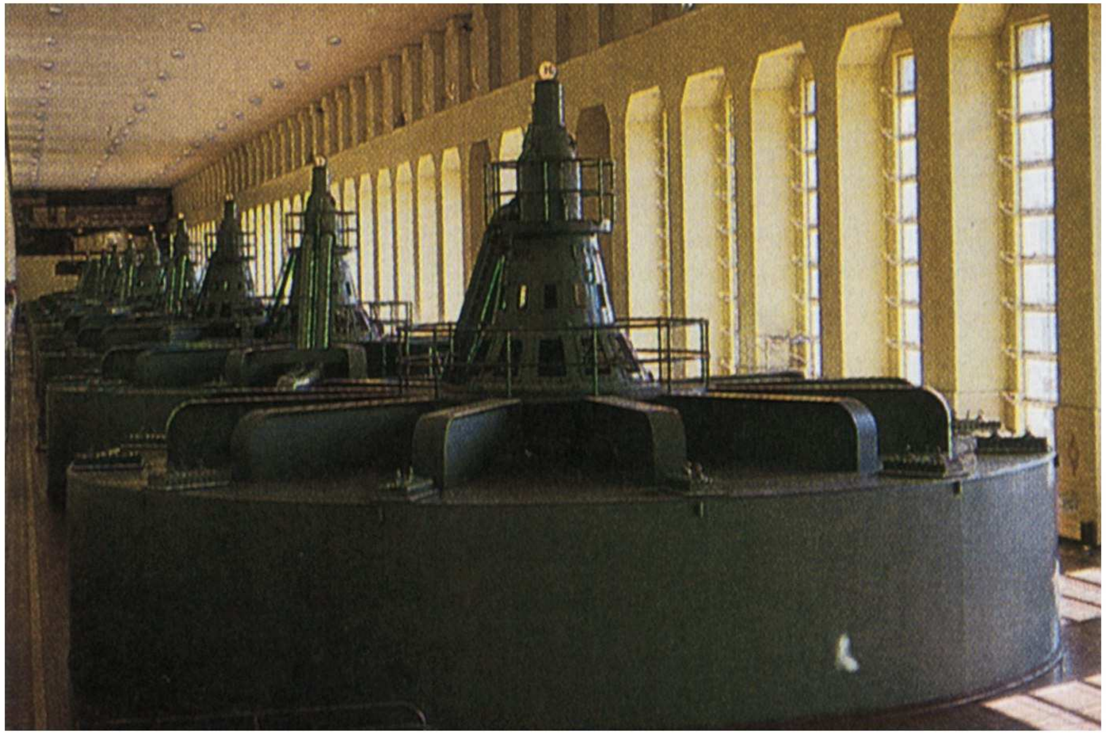
Generadors de corrent altern



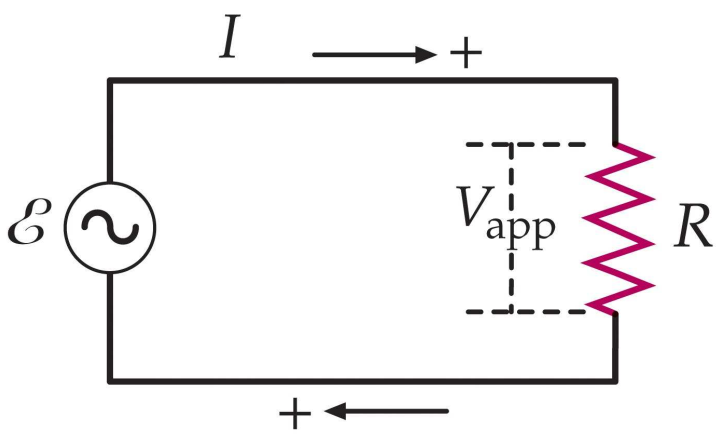


$$\phi_m = NBA\cos\theta = NBA\cos(\omega t + \delta) \qquad \omega = 2\pi\nu = \frac{2\pi}{T}$$

$$\mathcal{E} = -\frac{d\phi_m}{dt} = NBA\omega\sin(\omega t + \delta) = \mathcal{E}_{\max}\sin(\omega t + \delta)$$

Resistències en corrent altern

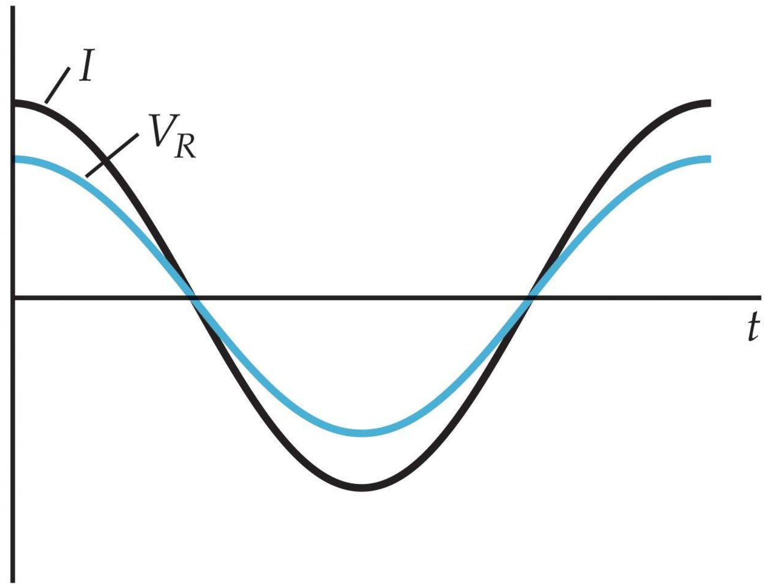


$$V_R = \mathcal{E} = \mathcal{E}_{\max} \sin(\omega t + \delta) \qquad \delta \to \frac{\pi}{2}$$

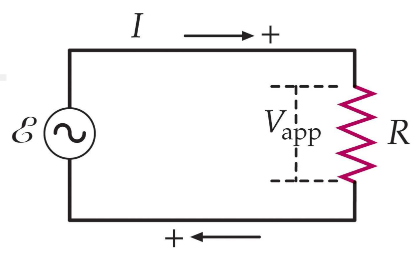
$$= V_{R,\max} \cos \omega t$$

$$I_R = \frac{V_R}{R} = \frac{V_{R,\max}}{R} \cos \omega t = I_{\max} \cos \omega t$$

Intensitat i potencial en fase

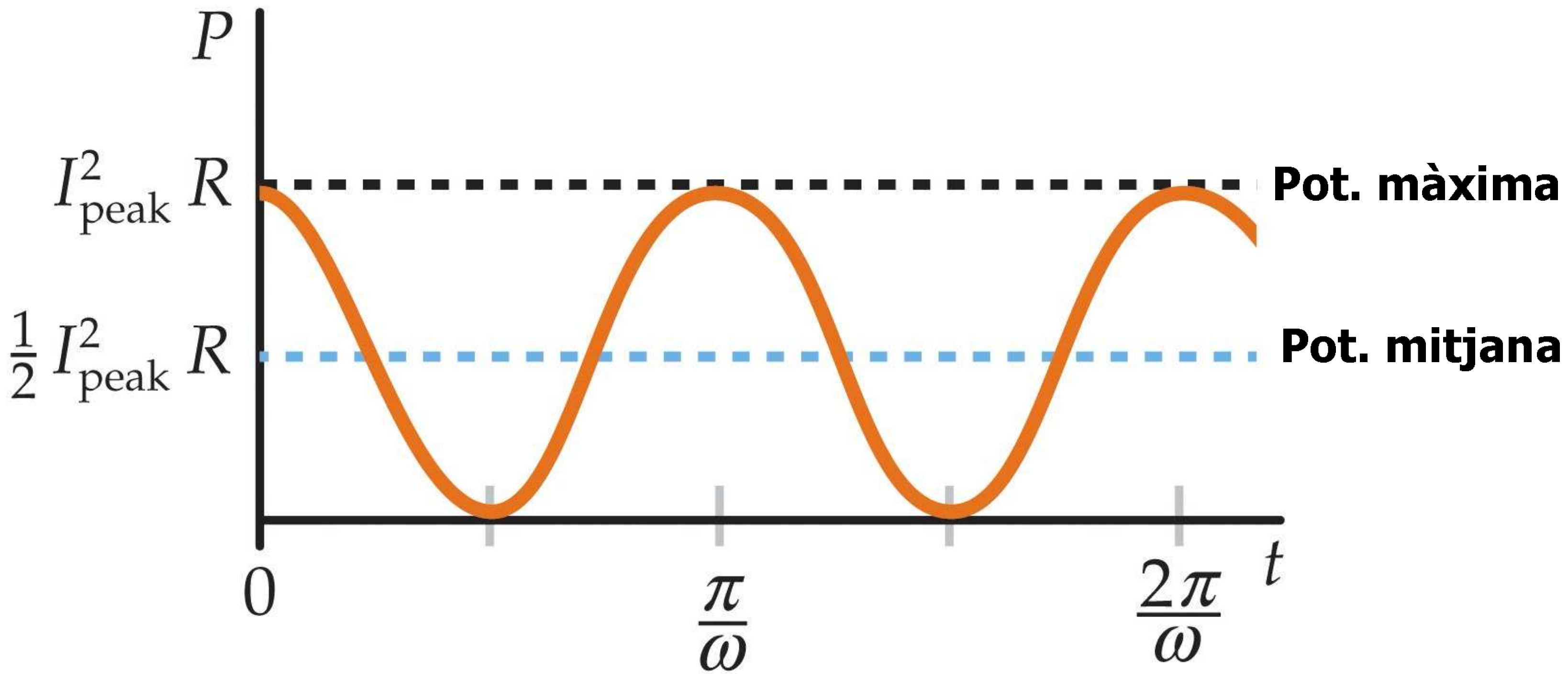


Potència

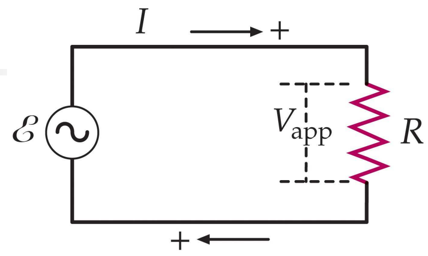


$$P = RI^2 = I_{max}^2 R \cos^2 \omega t$$

$$= P_{max} \cos^2 \omega t$$



Valors eficaços



$$I_{\text{ef}} \equiv \sqrt{(I^2)_m}$$

Valor mitjà

senyal sinusoidal

$$(I^2)_m = [(I_{\max} \cos \omega t)^2]_m = \frac{I_{\max}^2}{2}$$

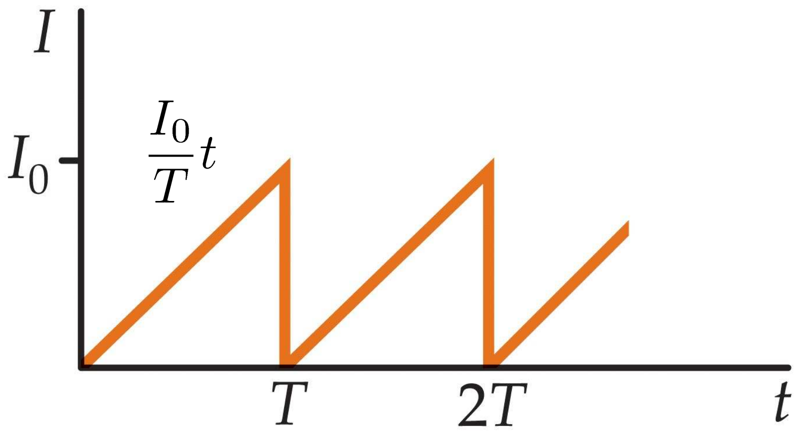
$$I_{\text{ef}} = \frac{I_{\max}}{\sqrt{2}} = 0.707 I_{\max}$$

$$P = I_{\max}^2 R \cos^2 \omega t \quad \Rightarrow \quad P_m = \frac{I_{\max}^2 R}{2} = I_{\text{ef}}^2 R$$

$$P_m = \mathcal{E}_{\max} I_{\max} (\cos^2 \omega t)_m = \frac{1}{2} \mathcal{E}_{\max} I_{\max}$$

$$P_m = \mathcal{E}_{\text{ef}} I_{\text{ef}}$$

senyal NO sinusoidal



$$I_m = \frac{1}{T}\int_0^T \frac{I_0}{T} t\,dt = \frac{I_0}{2}$$

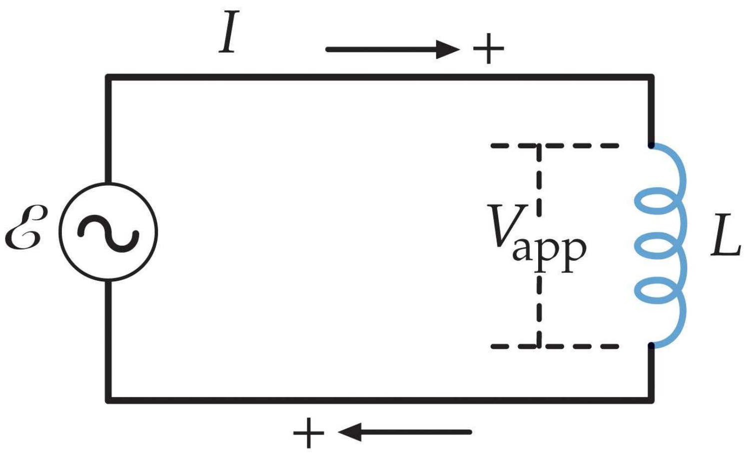
$$(I^2)_m = \frac{1}{T}\int_0^T \frac{I_0^2}{T^2} t^2\,dt = \frac{I_0^2}{3}$$

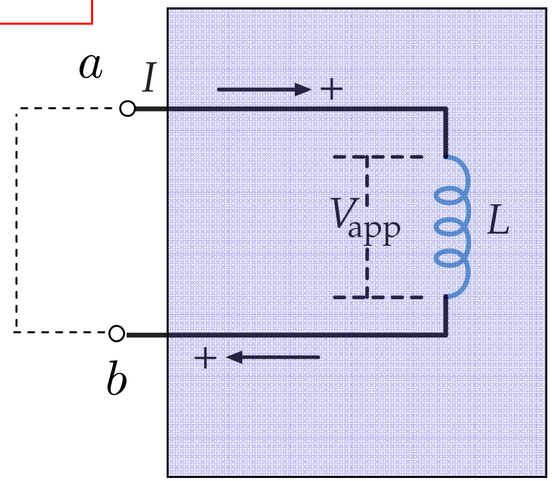
$$I_{\text{ef}} \equiv \sqrt{(I^2)_m} = \frac{I_0}{\sqrt{3}}$$

$$P_m = R I_{\text{ef}}^2$$

Bobines en corrent altern

$$\boxed{V_L = L\frac{dI}{dt}}$$



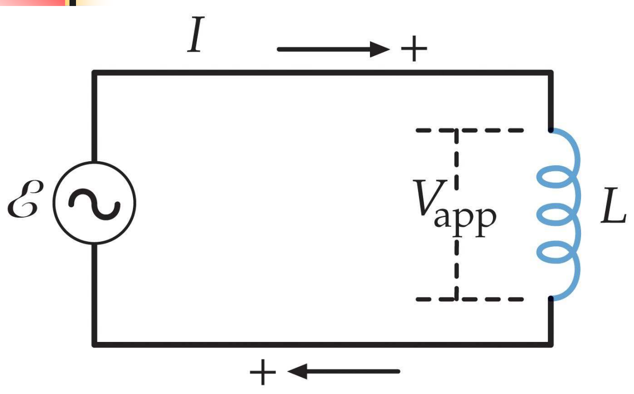


$$\left.\begin{array}{l} \displaystyle\oint_C \vec{E}\cdot d\vec{\ell} = -L\frac{dI}{dt} \\ \displaystyle\oint_C \vec{E}\cdot d\vec{\ell} = \underbrace{\int_a^b \vec{E}\cdot d\vec{\ell}}_{\text{Bobina}} + \underbrace{\int_b^a \vec{E}\cdot d\vec{\ell}}_{\text{Fora}} = V_b - V_a \end{array}\right\} \quad V_a - V_b = L\frac{dI}{dt}$$

$= 0$

Fils conductors ideals

No hi ha camps magnètics variables
Es pot definir el potencial elèctric

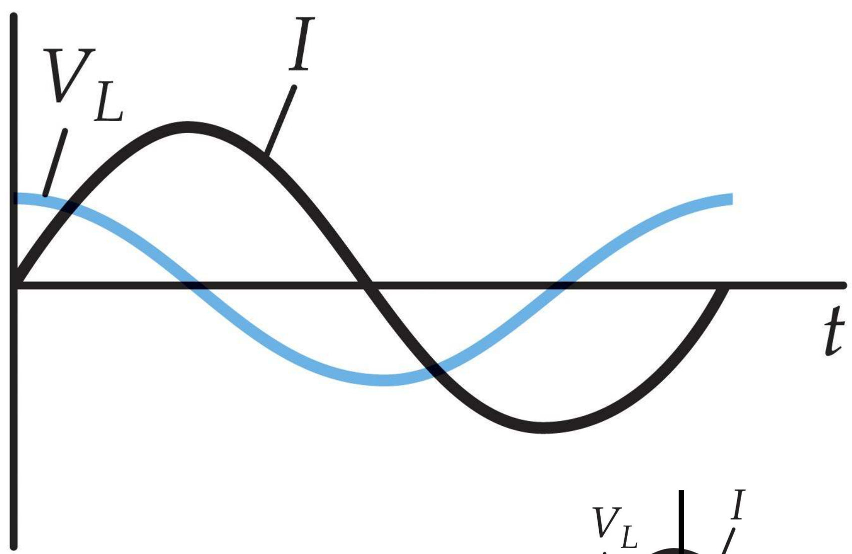


$$V_L = \mathcal{E} = V_{L,\max}\cos\omega t$$

$$V_L = L\frac{dI}{dt} \Longrightarrow I = \underbrace{\frac{V_{L,\max}}{\omega L}}_{I_{\max}}\sin\omega t$$

$$I = I_{\max}\cos(\omega t - \pi/2)$$

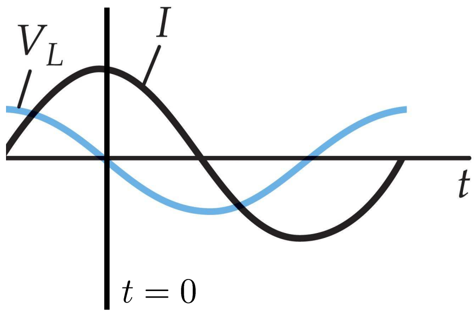
Desfassament
Potencial adelantat

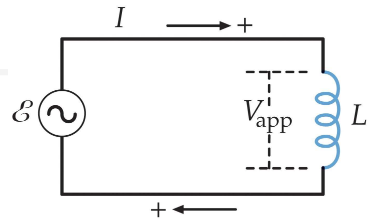


Equivalentment

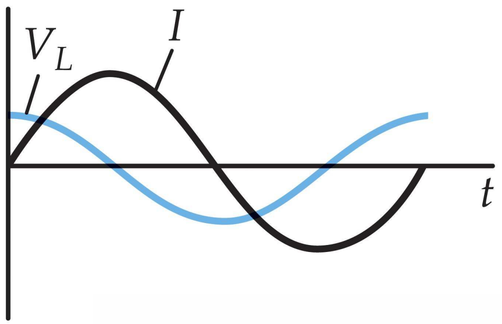
$$I = I_{\max}\,\cos(\omega t)$$

$$V_L = V_{L,\max}\cos(\omega t + \pi/2)$$





$$I = \frac{V_{L,\max}}{\omega L} \sin \omega t$$



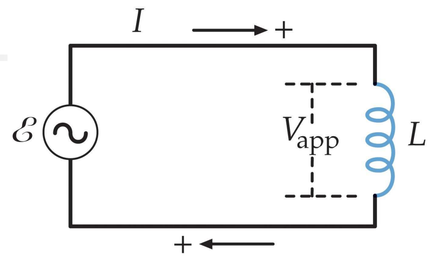
$$I_{\max} = \frac{V_{L,\max}}{\omega L} = \frac{V_{L,\max}}{X_L}$$

$$X_L = \omega L \qquad 29\text{-}24$$

DEFINITION—INDUCTIVE REACTANCE

També $I_{\text{ef}} = \dfrac{V_{L,\text{ef}}}{X_L}$

Potència

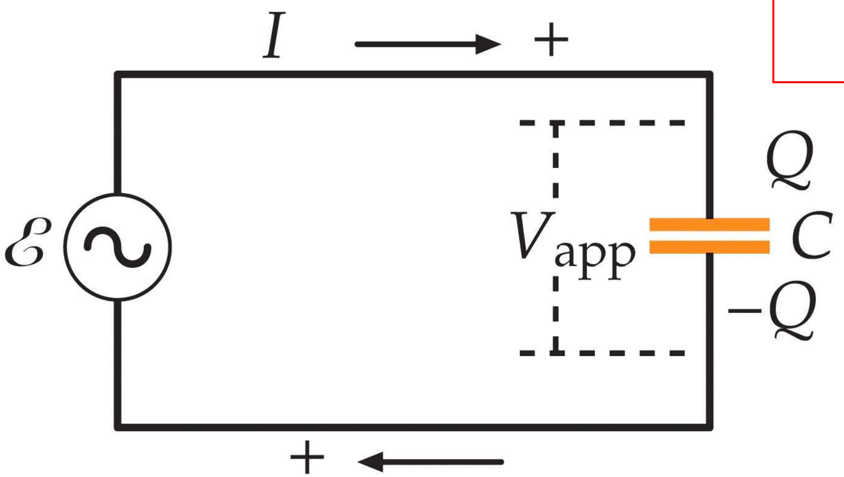


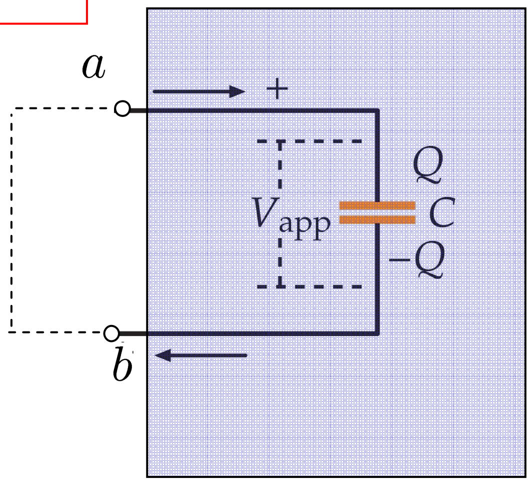
$$P \equiv V_L I = V_{L,\max} I_{\max} \underbrace{\cos\omega t \ \sin\omega t}_{\frac{1}{2}\sin 2\omega t}$$

$$\boxed{P_m = 0}$$

Condensadors en corrent altern

$$V_C = \frac{Q}{C}$$



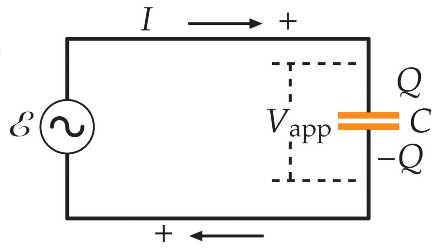


$$V_a - V_b = \frac{Q}{C}$$

$$V_C = \mathcal{E} = V_{C,\max} \cos \omega t$$

$$Q = V_C C = V_{C,\max} C \cos \omega t$$

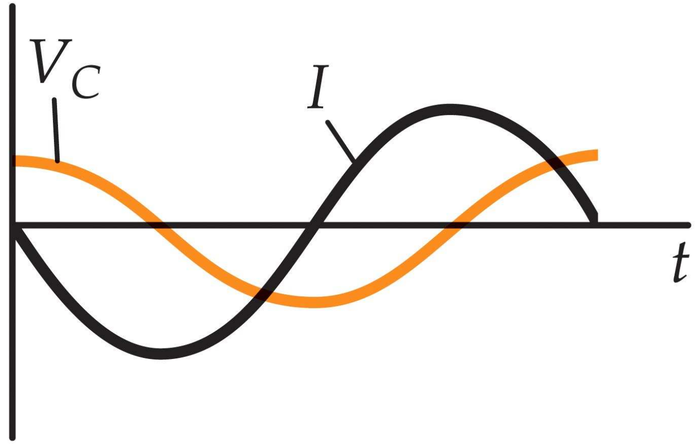
$$I = \frac{dQ}{dt} = \underbrace{-\omega V_{C,\max} C}_{I_{\max}} \sin \omega t = I_{\max} \cos(\omega t + \pi/2)$$



$$V_C = \mathcal{E} = V_{C,\max} \cos \omega t$$

$$I = -\omega V_{C,\max} C \sin \omega t = I_{\max} \cos(\omega t + \pi/2)$$

Desfassament
Potencial retrassat

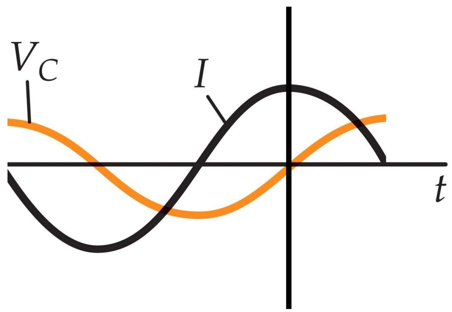


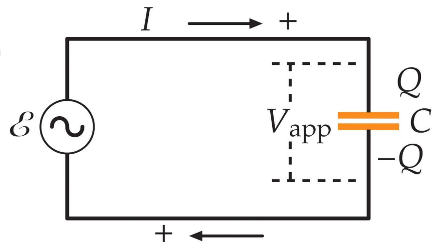
$$I_{\max} = \omega\, C\, V_{C,\max} = \frac{V_{C,\max}}{\frac{1}{\omega C}} \equiv \frac{V_{C,\max}}{X_C}$$

Equivalentement

$$I = I_{\max} \cos(\omega t)$$

$$V_C = V_{C,\max} \cos(\omega t - \pi/2)$$





$$X_C = \frac{1}{\omega C} \qquad 29\text{-}30$$

DEFINITION—CAPACITIVE REACTANCE

$$I_{\max} = \frac{V_{C,\max}}{X_C}$$

$$I_{\mathrm{ef}} = \frac{V_{C,\mathrm{ef}}}{X_C}$$

$$P \equiv V_C I = -V_{C,\max} I_{\max} \cos\omega t \, \sin\omega t \;\Rightarrow\; P_m = 0$$

Representació amb nombres complexos

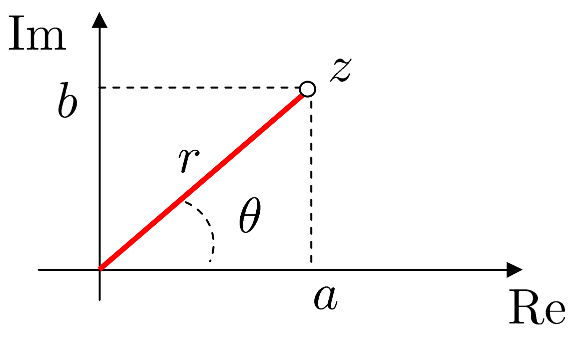
Fórmula d'Euler $$e^{i\theta} = \cos\theta + i\sin\theta \qquad \begin{cases} \cos\theta = \mathrm{Re}\left(e^{i\theta}\right) \\ \sin\theta = \mathrm{Im}\left(e^{i\theta}\right) \end{cases}$$

Nombre complexe $$z = a + ib = re^{i\theta}$$

mòdul $$r = \sqrt{a^2 + b^2}$$

fase $$\tan\theta = \frac{b}{a}$$

Pla complexe



$$V = V_{\max}\cos(\omega t + \varphi_0) = \mathrm{Re}\left(V_{\max} e^{i\varphi_0} e^{i\omega t}\right)$$

$$\hat{V} = V_{\max} e^{i\varphi_0}$$ amplada complexe (o fasor)

$$\left.\begin{aligned} V &= \mathrm{Re}\left(\hat{V} e^{i\omega t}\right) \\ I &= \mathrm{Re}\left(\hat{I} e^{i\omega t}\right) \\ &\ldots \end{aligned}\right\} \text{fase temporal comuna } e^{i\omega t}$$

A vegades, abús de notació

$$V = \hat{V} e^{i\omega t}$$

$$I = \hat{I} e^{i\omega t}$$

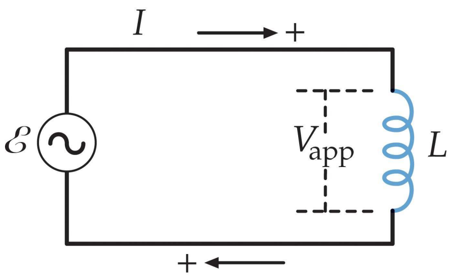
Impedàncies complexes

$$Z = \frac{\hat{V} e^{i\omega t}}{\hat{I} e^{i\omega t}} = \frac{\hat{V}}{\hat{I}}$$

la fase de Z és el desfassament entre potencial i intensitat

$$\varphi > 0 \Rightarrow V \text{ adelantat}$$
$$\varphi < 0 \Rightarrow V \text{ retrassat}$$

bobina

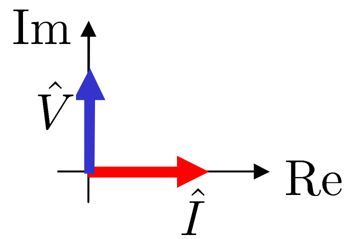


$$V = V_{\max} \cos \omega t = \mathrm{Re}\left(V_{\max} e^{i\omega t}\right)$$

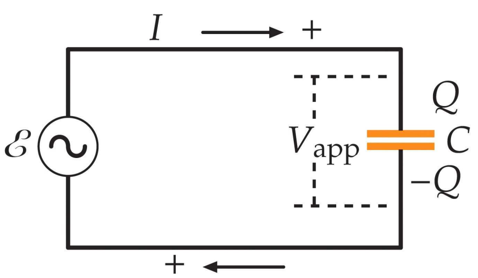
$$I = \frac{V_{\max}}{\omega L} \cos(\omega t - \pi/2) = \mathrm{Re}\left(\frac{V_{\max}}{\omega L} e^{-i\pi/2} e^{i\omega t}\right)$$

$$e^{-i\pi/2} = -i = \frac{1}{i}$$

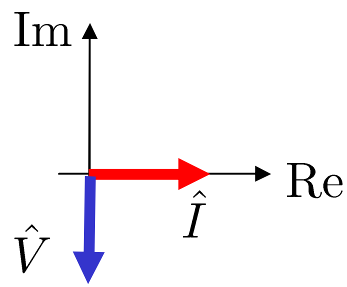
$$Z = \frac{\hat{V}}{\hat{I}} = i\omega L$$



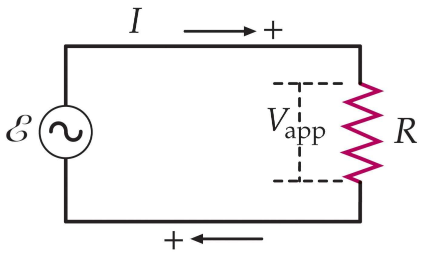
condensador



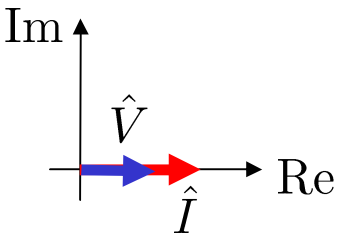
$$Z = \frac{\hat{V}}{\hat{I}} = \frac{1}{i\omega C} = -i \frac{1}{\omega C}$$



Resistència



$$Z = \frac{\hat{V}}{\hat{I}} = R$$



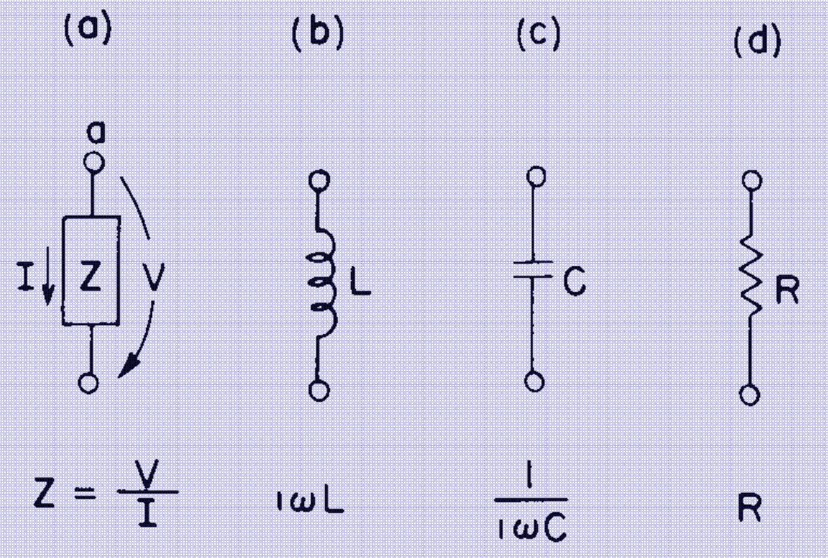


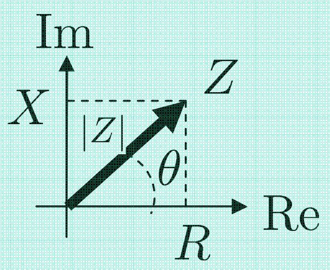
Fig. 22–4. The ideal lumped circuit elements (passive).

Impedància general

$$Z = R + iX$$

$$|Z| = \sqrt{R^2 + X^2}$$

$$\tan\theta = \frac{X}{R}$$



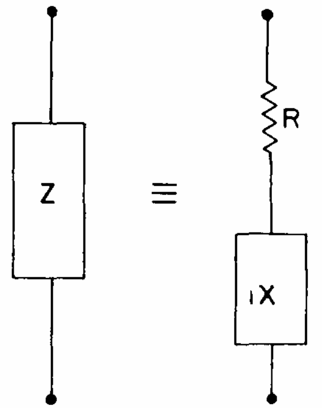
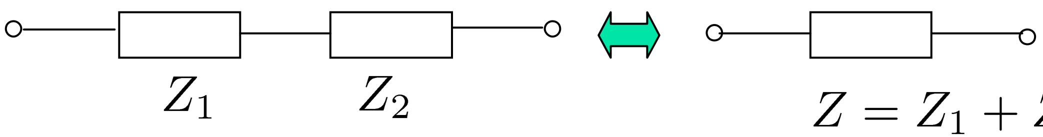


Fig. 22–17. Any impedance is equivalent to a series combination of a pure resistance and a pure reactance.

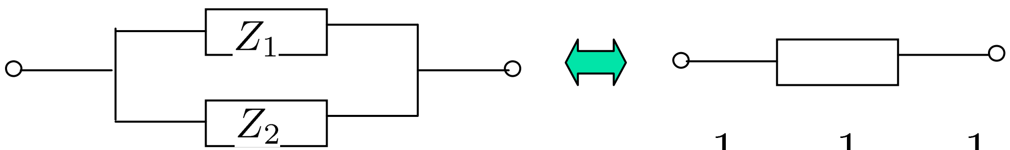
Associació d'impedàncies

Sèrie



$$Z = Z_1 + Z_2$$

Paral.lel



$$\frac{1}{Z} = \frac{1}{Z_1} + \frac{1}{Z_2}$$

Lleis de Kirchoff

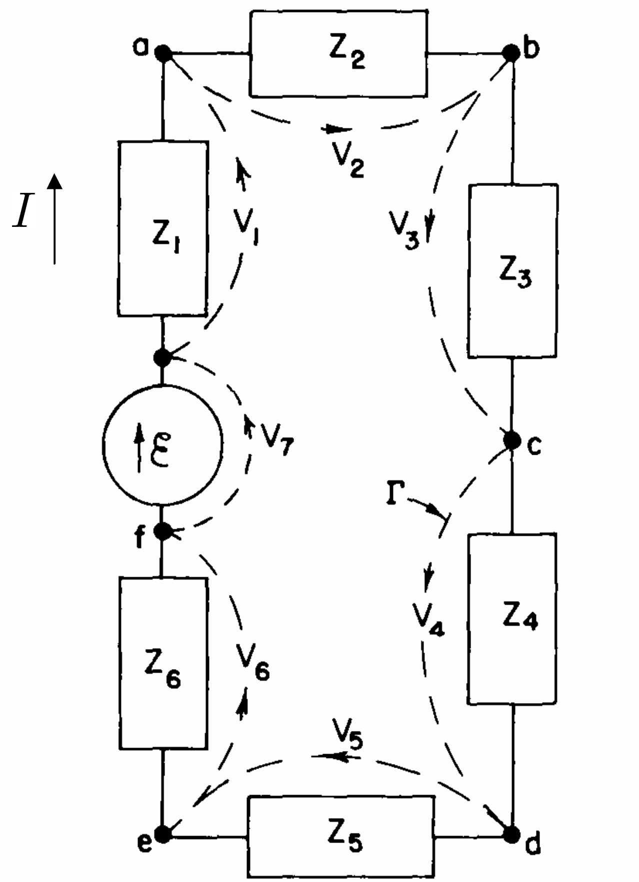


Fig. 22–9. The sum of the voltage drops around any closed path is zero.

$$-Z_1\hat{I} - Z_2\hat{I} - Z_3\hat{I} - Z_4\hat{I} - Z_5\hat{I} + Z_6\hat{I} + \hat{\mathcal{E}} = 0$$

$$\hat{I} = \frac{\hat{\mathcal{E}}}{\sum_i Z_i} \qquad I = \mathrm{Re}\left(\hat{I}\, e^{i\omega t}\right)$$

$$\frac{\hat{\mathcal{E}}}{\hat{I}} = \sum_i Z_i$$

Potència a partir de la impedancia

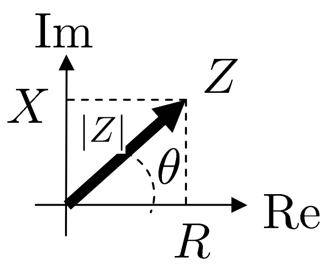
$$P = VI$$

$$\hat{V} = Z\hat{I} = (R + iX)\hat{I} \quad \Longrightarrow \quad V = \mathrm{Re}\left((R + iX)I_0 e^{i\omega t}\right)$$
$$= RI_0 \cos\omega t - XI_0 \sin\omega t$$

$$P = VI = RI_0^2 \cos^2\omega t - XI_0^2 \sin\omega t \cos\omega t \quad \Longrightarrow \quad P_m = R\frac{I_0^2}{2}$$

$$\boxed{P_m = RI_e^2}$$

però $R = |Z| \cos\theta$



també $\hat{V} = Z\hat{I} \quad \Longrightarrow \quad |\hat{V}| = |Z||\hat{I}|$

$$V_0 = |Z|I_0$$

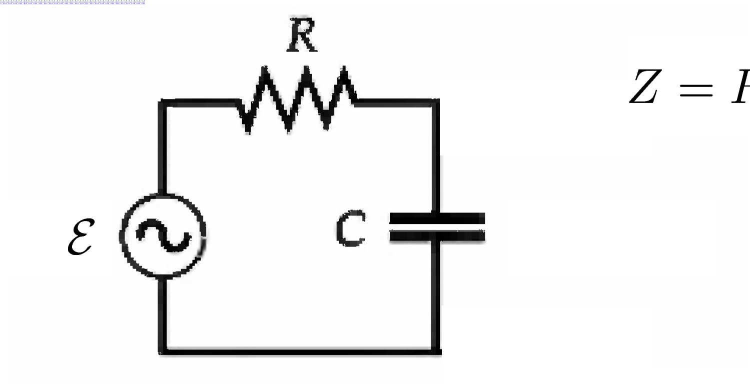
$$P_m = \frac{V_0 I_0}{2} \cos\theta$$

$$\boxed{P_m = V_e I_e \cos\theta}$$

Factor de potència

Circuits simples

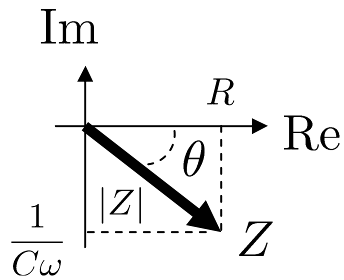
RC



$$Z = R + \frac{1}{iC\omega} = R - i\frac{1}{C\omega}$$

$$|Z| = \sqrt{R^2 + \frac{1}{C^2\omega^2}}$$

$$\theta = -\mathrm{atan}\left(\frac{\frac{1}{C\omega}}{R}\right)$$



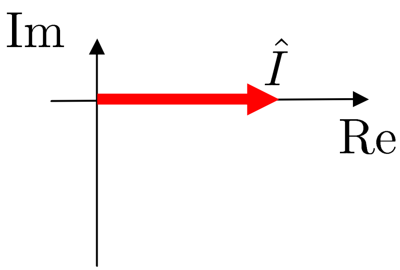
$$\hat{I} = \frac{\hat{\mathcal{E}}}{Z}$$

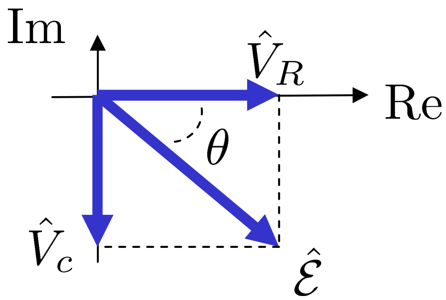
$$I = \mathrm{Re}\left(\hat{I}\, e^{i\omega t}\right)$$

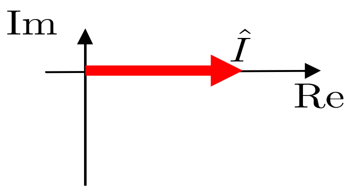
$$\hat{V}_R = R\hat{I}$$

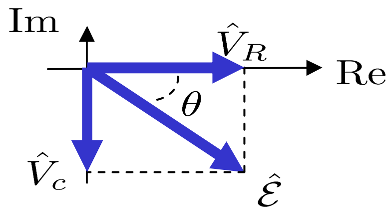
$$\hat{V}_C = \frac{1}{iC\omega}\hat{I}$$

$$\hat{\mathcal{E}} = \hat{V}_R + \hat{V}_C$$









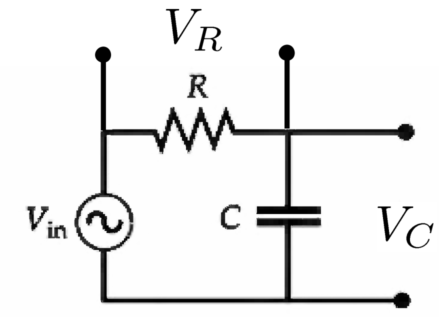
$$I = I_{\max} \cos \omega t \qquad I_{\max} = \frac{\mathcal{E}_{\max}}{|Z|} = \frac{\mathcal{E}_{\max}}{\sqrt{R^2 + \frac{1}{C^2\omega^2}}}$$

$$\mathcal{E} = \mathcal{E}_{\max} \cos(\omega t - \theta) \qquad \mathcal{E}_{\max}$$

$$V_R = V_{R,\max} \cos(\omega t) \qquad V_{R,\max} = R I_{\max} = \frac{R\mathcal{E}_{\max}}{\sqrt{R^2 + \frac{1}{C^2\omega^2}}}$$

$$V_C = V_{C,\max} \cos(\omega t - \pi/2) \qquad V_{C,\max} = \frac{1}{C\omega} I_{\max} = \frac{\frac{1}{C\omega}\mathcal{E}_{\max}}{\sqrt{R^2 + \frac{1}{C^2\omega^2}}}$$

Filtres



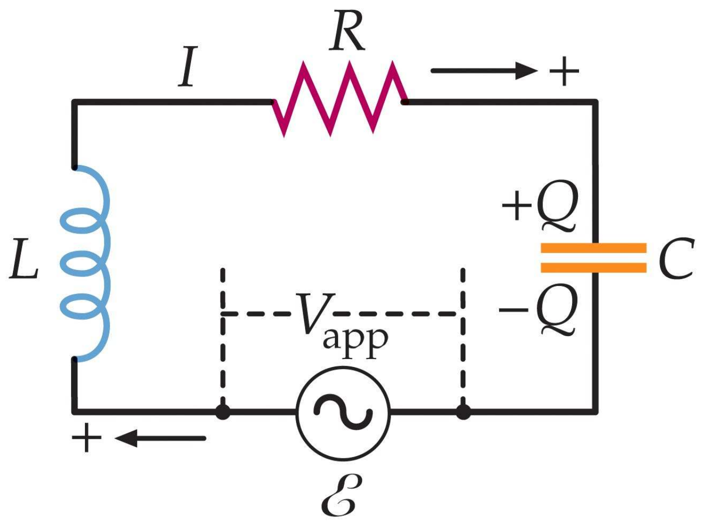
$$\omega \to 0 \quad \begin{cases} V_{R,\max} \to 0 \\ V_{C,\max} \to \mathcal{E}_{\max} \end{cases}$$ Passa baixa

$$\omega \to \infty \quad \begin{cases} V_{R,\max} \to \mathcal{E}_{\max} \\ V_{C,\max} \to 0 \end{cases}$$ Passa alta

RL

Exercici

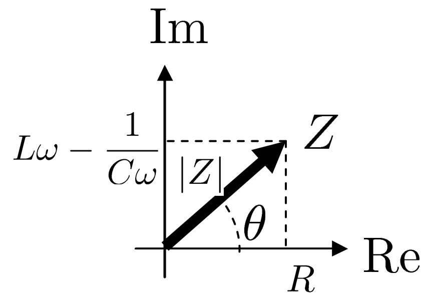
RCL



$$Z = R + i\left(L\omega - \frac{1}{C\omega}\right)$$

$$|Z| = \sqrt{R^2 + \left(L\omega - \frac{1}{C\omega}\right)^2}$$

$$\theta = \operatorname{atan}\left(\frac{L\omega - \frac{1}{C\omega}}{R}\right)$$



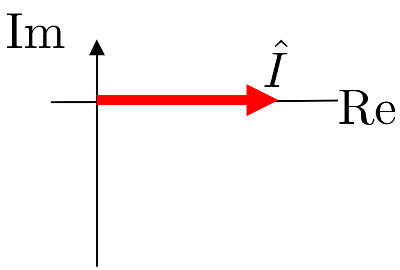
$$\hat{I} = \frac{\hat{\mathcal{E}}}{Z}$$

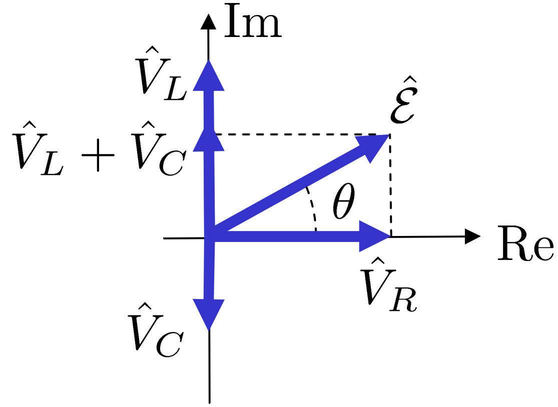
$$\hat{V}_R = R\hat{I}$$

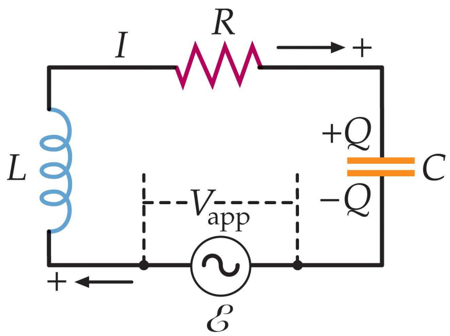
$$\hat{V}_C = \frac{1}{iC\omega}\hat{I}$$

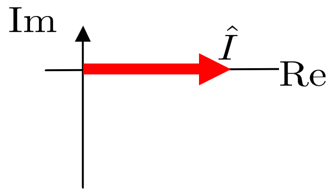
$$\hat{V}_L = iL\omega\hat{I}$$

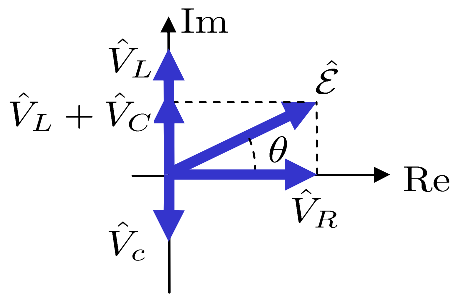
$$\hat{\mathcal{E}} = \hat{V}_R + \hat{V}_C + \hat{V}_L$$











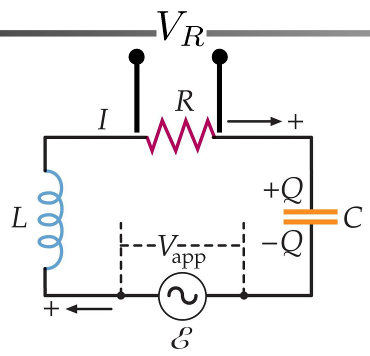
$$I = I_{\max} \cos \omega t \qquad I_{\max} = \frac{\mathcal{E}_{\max}}{|Z|} = \frac{\mathcal{E}_{\max}}{\sqrt{R^2 + \left(L\omega - \frac{1}{C\omega}\right)^2}}$$

$$\mathcal{E} = \mathcal{E}_{\max} \cos(\omega t + \theta) \qquad \mathcal{E}_{\max}$$

$$V_R = V_{R,\max} \cos(\omega t) \qquad V_{R,\max} = R I_{\max} = \frac{R\mathcal{E}_{\max}}{\sqrt{R^2 + \left(L\omega - \frac{1}{C\omega}\right)^2}}$$

$$V_C = V_{C,\max} \cos(\omega t - \pi/2) \qquad V_{C,\max} = \frac{1}{C\omega} I_{\max} = \frac{\frac{1}{C\omega}\mathcal{E}_{\max}}{\sqrt{R^2 + \left(L\omega - \frac{1}{C\omega}\right)^2}}$$

$$V_L = V_{L,\max} \cos(\omega t + \pi/2) \qquad V_{L,\max} = i L\omega I_{\max} = \frac{L\omega\mathcal{E}_{\max}}{\sqrt{R^2 + \left(L\omega - \frac{1}{C\omega}\right)^2}}$$



$$V_{R,\max} = RI_{\max} = \frac{R\mathcal{E}_{\max}}{\sqrt{R^2 + \left(L\omega - \frac{1}{C\omega}\right)^2}}$$

$$\begin{cases} \omega \to 0 & V_{R,\max} \to 0 \\ \omega = \sqrt{\dfrac{1}{LC}} & V_{R,\max} = \mathcal{E}_{\max} \\ \omega \to \infty & V_{R,\max} \to 0 \end{cases}$$

Filtre
"passa banda"

Potència

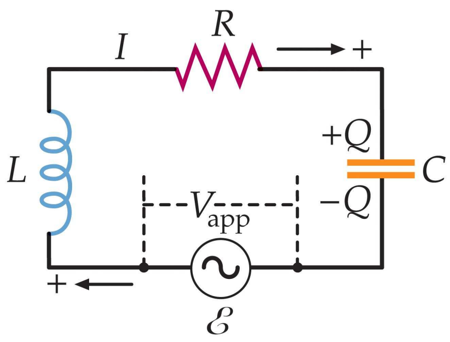
$$P_m = R\frac{I_{\max}^2}{2} = \frac{R}{2}\frac{\mathcal{E}_{\max}^2}{R^2 + \left(L\omega - \frac{1}{C\omega}\right)^2}$$

$$= \frac{R\mathcal{E}_{\max}^2}{2}\frac{\omega^2}{L^2(\omega^2 - \omega_0^2) + \omega^2 R^2}$$

$$\omega_0 = \sqrt{\frac{1}{LC}}$$

Ressonància a $\omega = \omega_0$

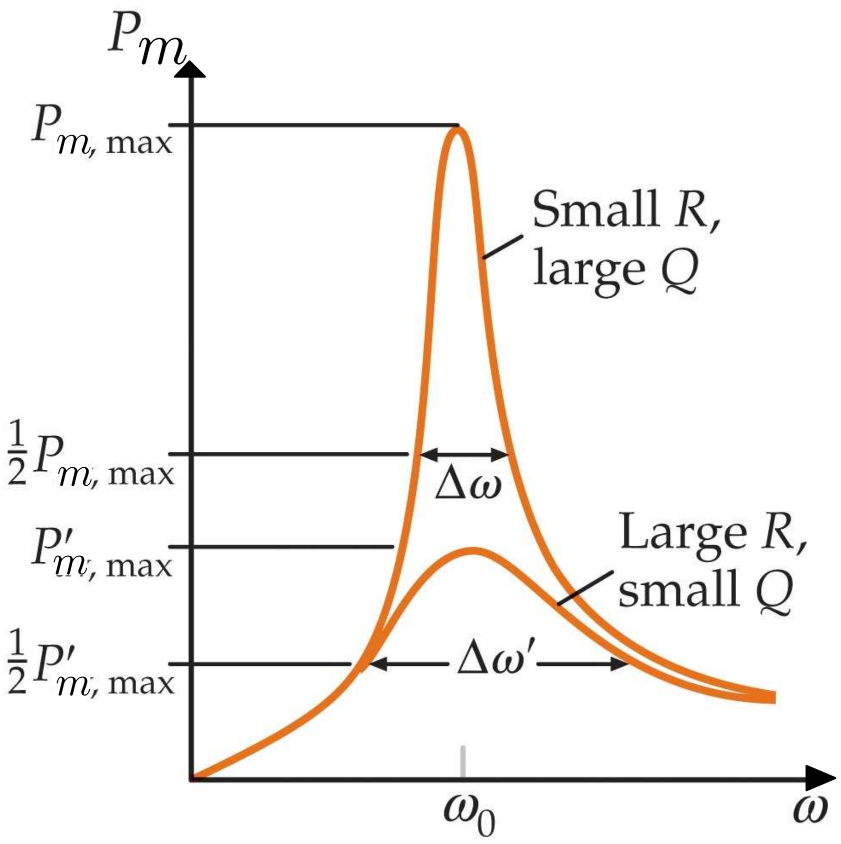
$$P_m = \frac{R\mathcal{E}_{\max}^2}{2} \frac{\omega^2}{L^2(\omega^2 - \omega_0^2) + \omega^2 R^2}$$



$$\omega_0 = \sqrt{\frac{1}{LC}}$$

$$Q = \frac{\omega_0 L}{R}$$

$$\Delta\omega \approx \frac{\omega_0}{Q}$$



Problems

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29.5

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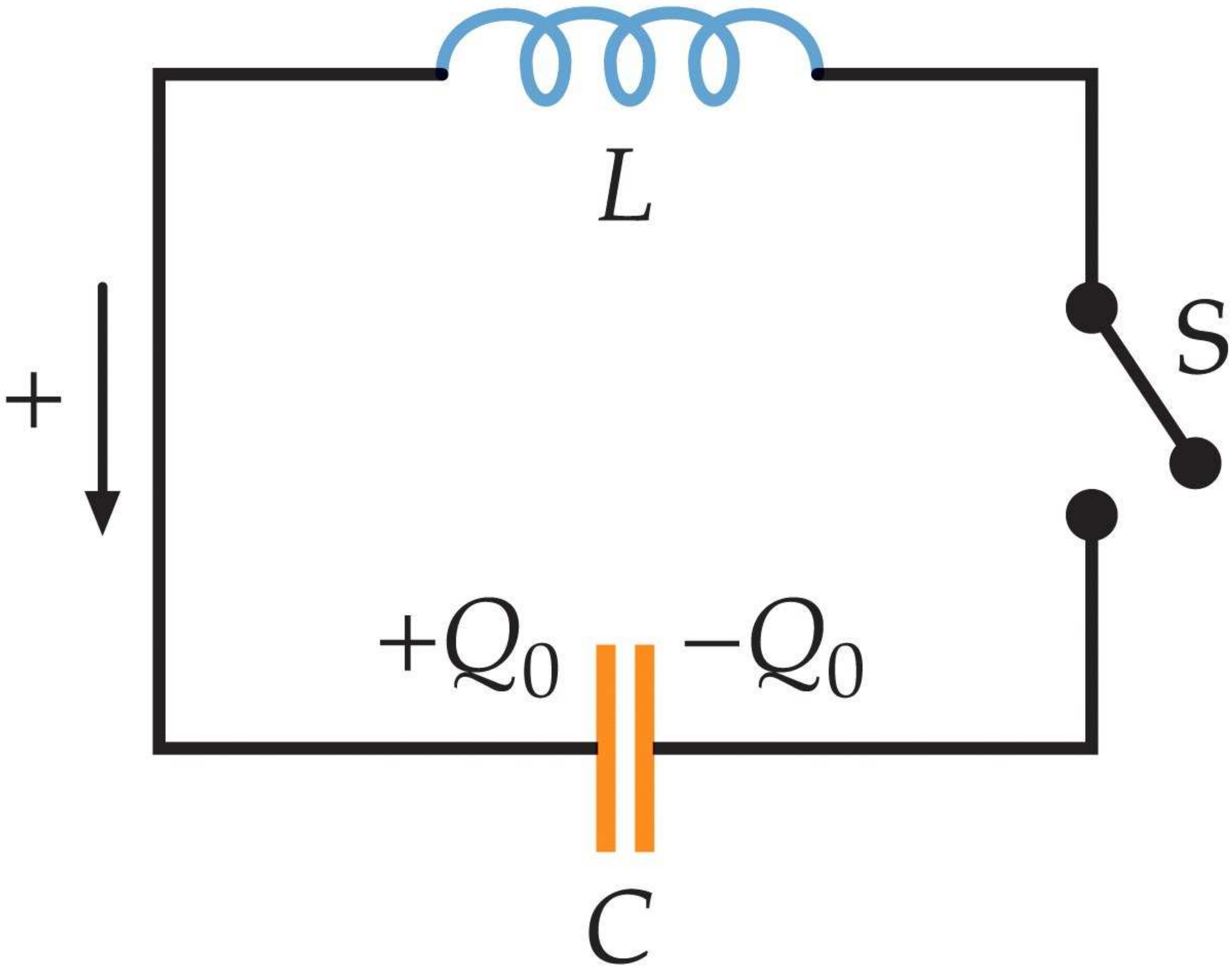

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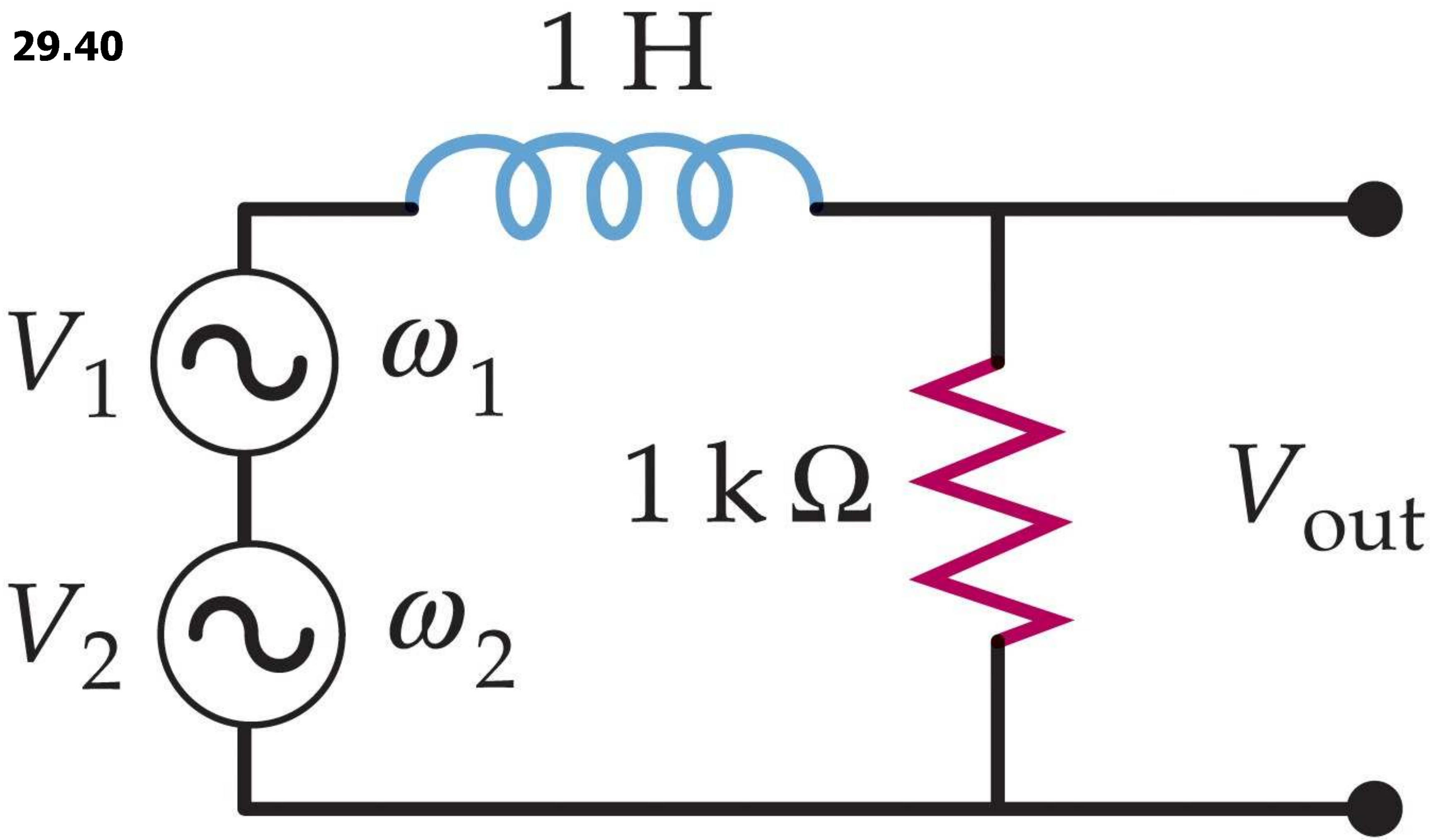

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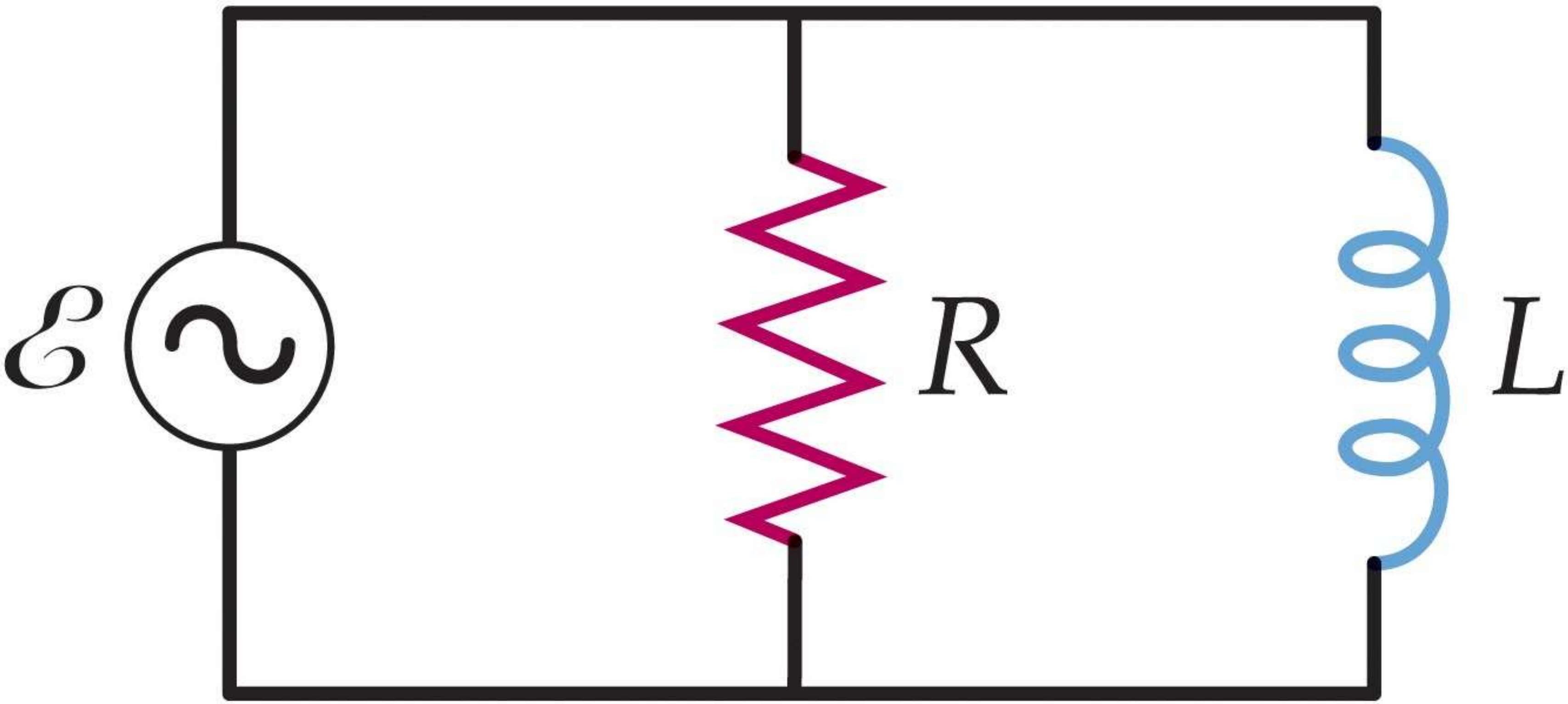

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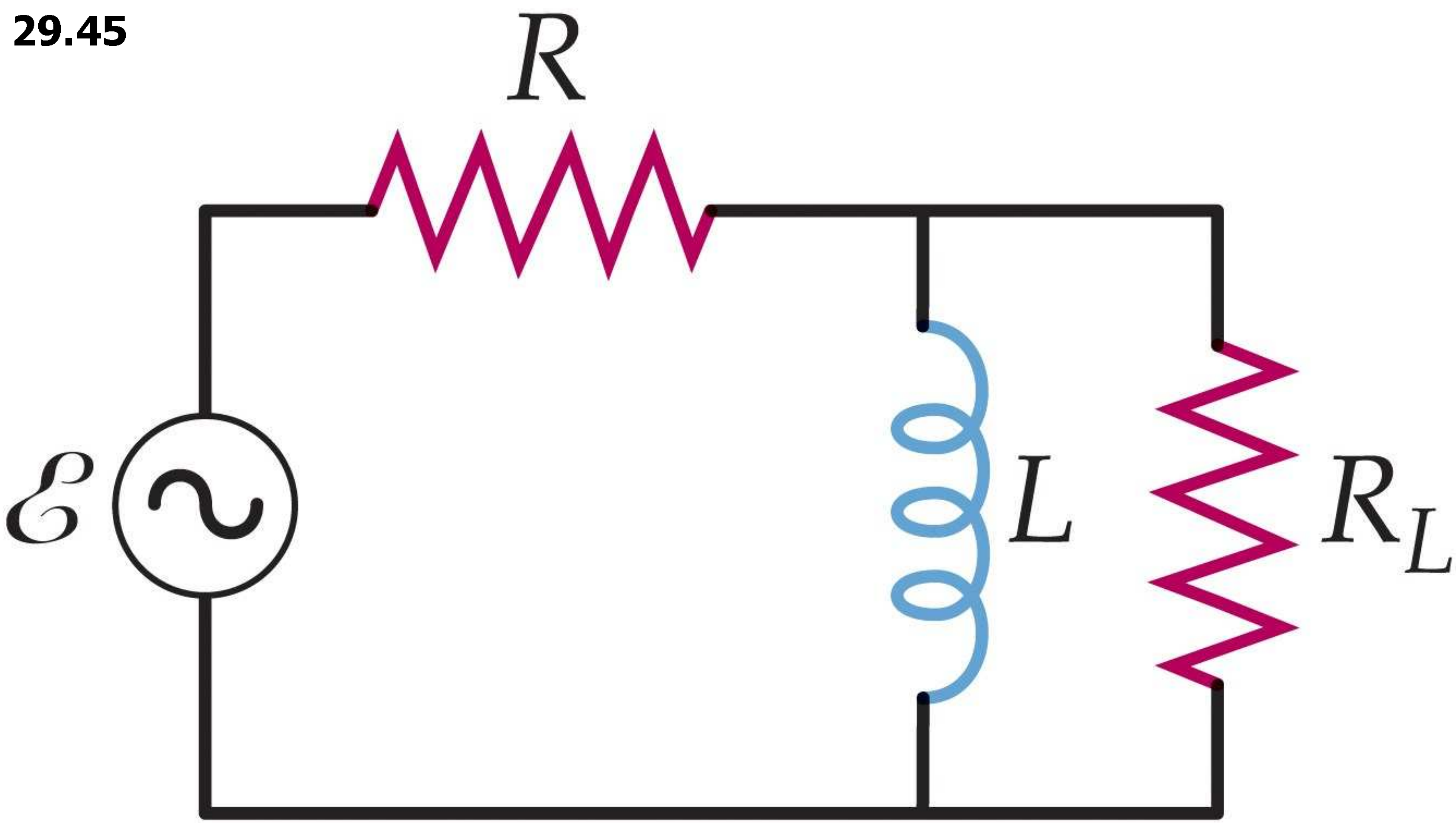

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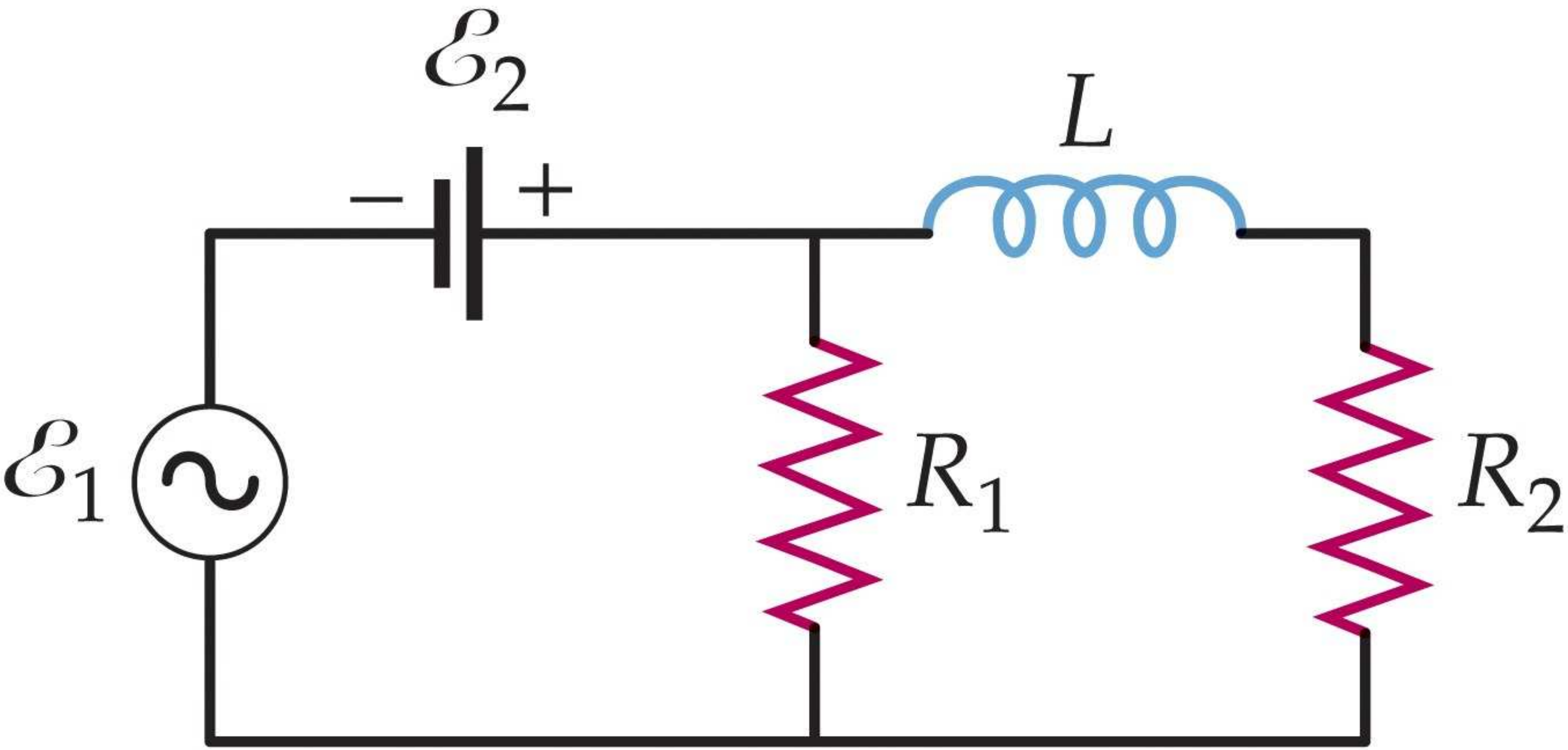

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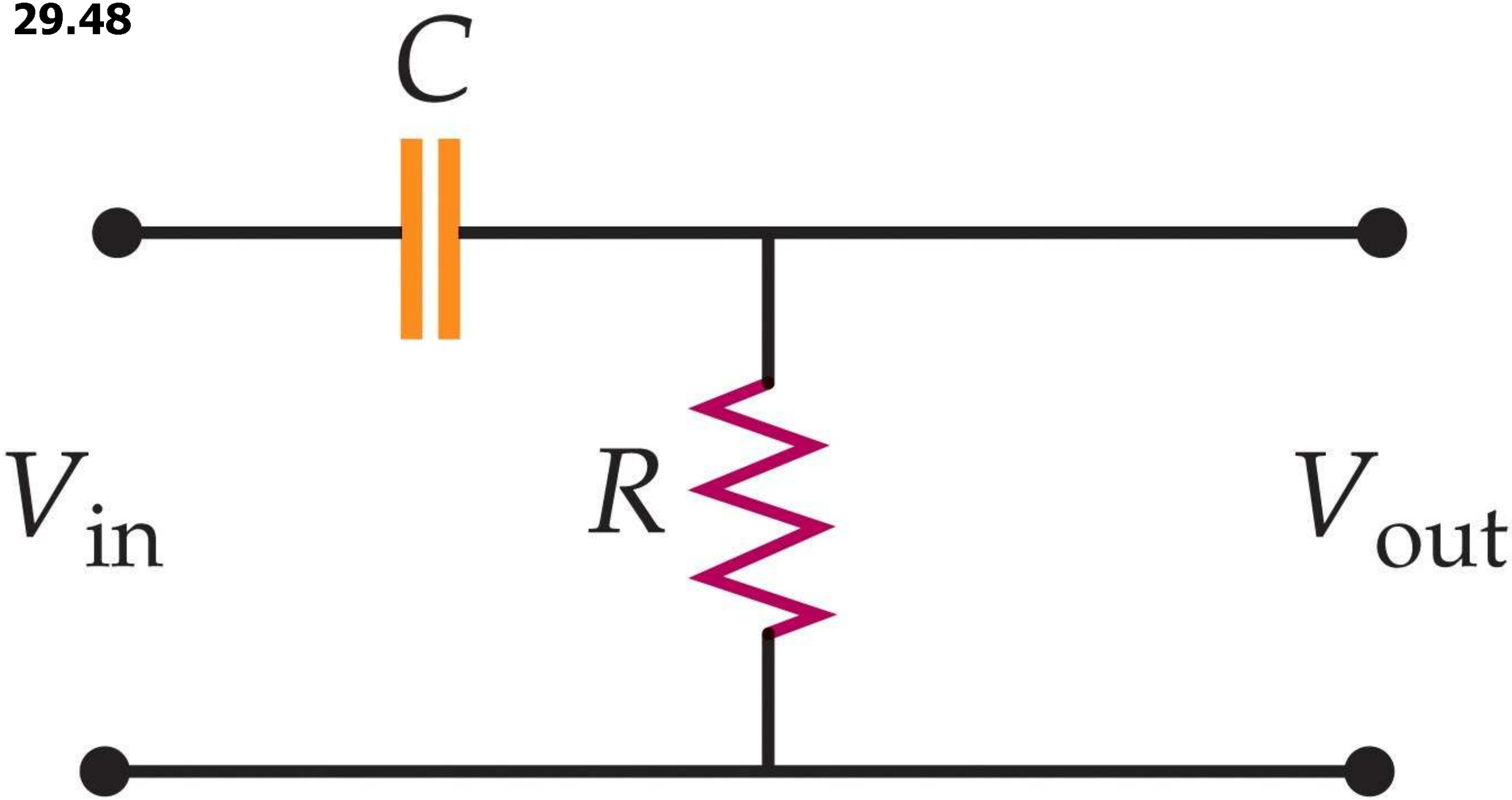

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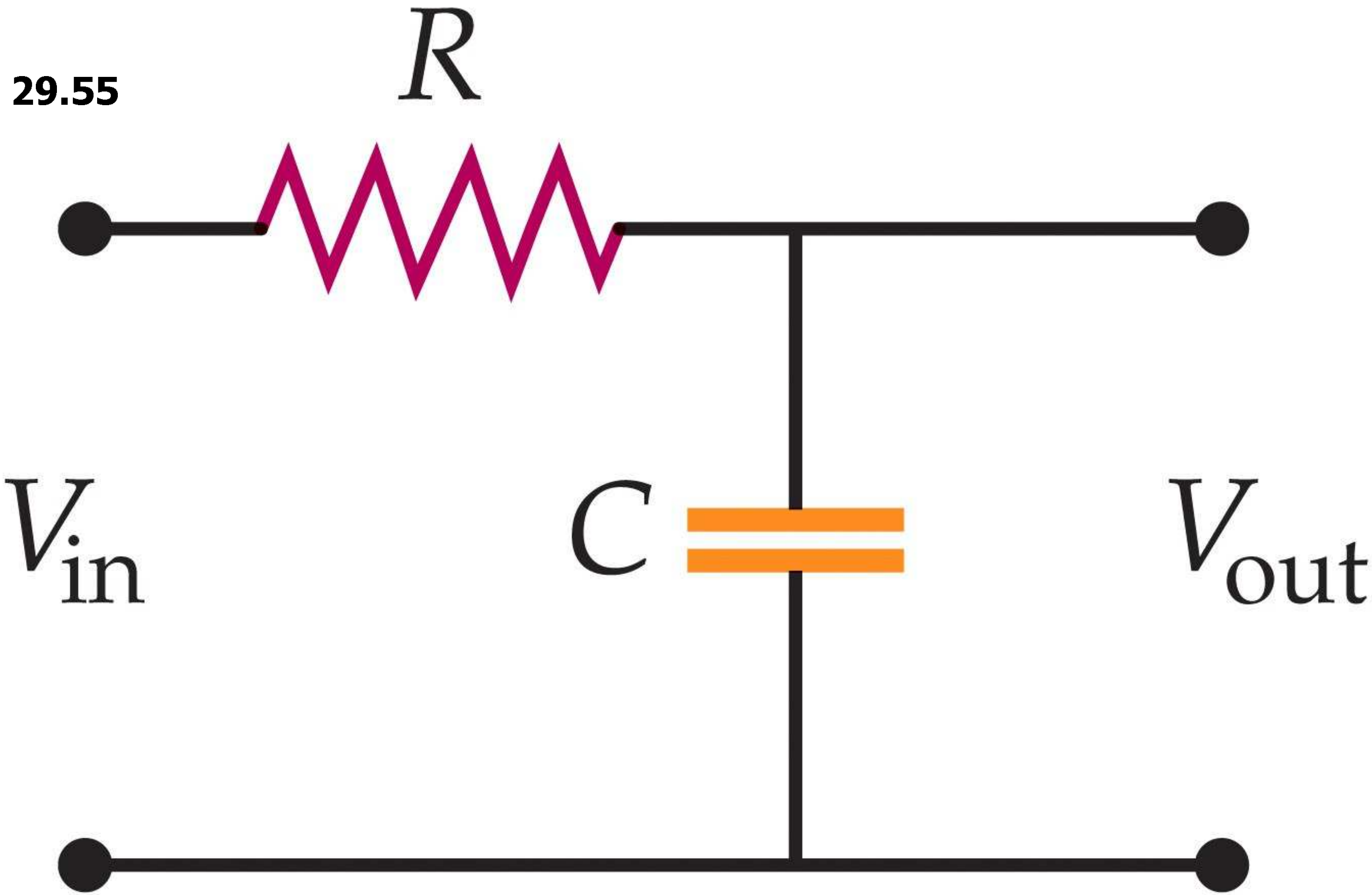

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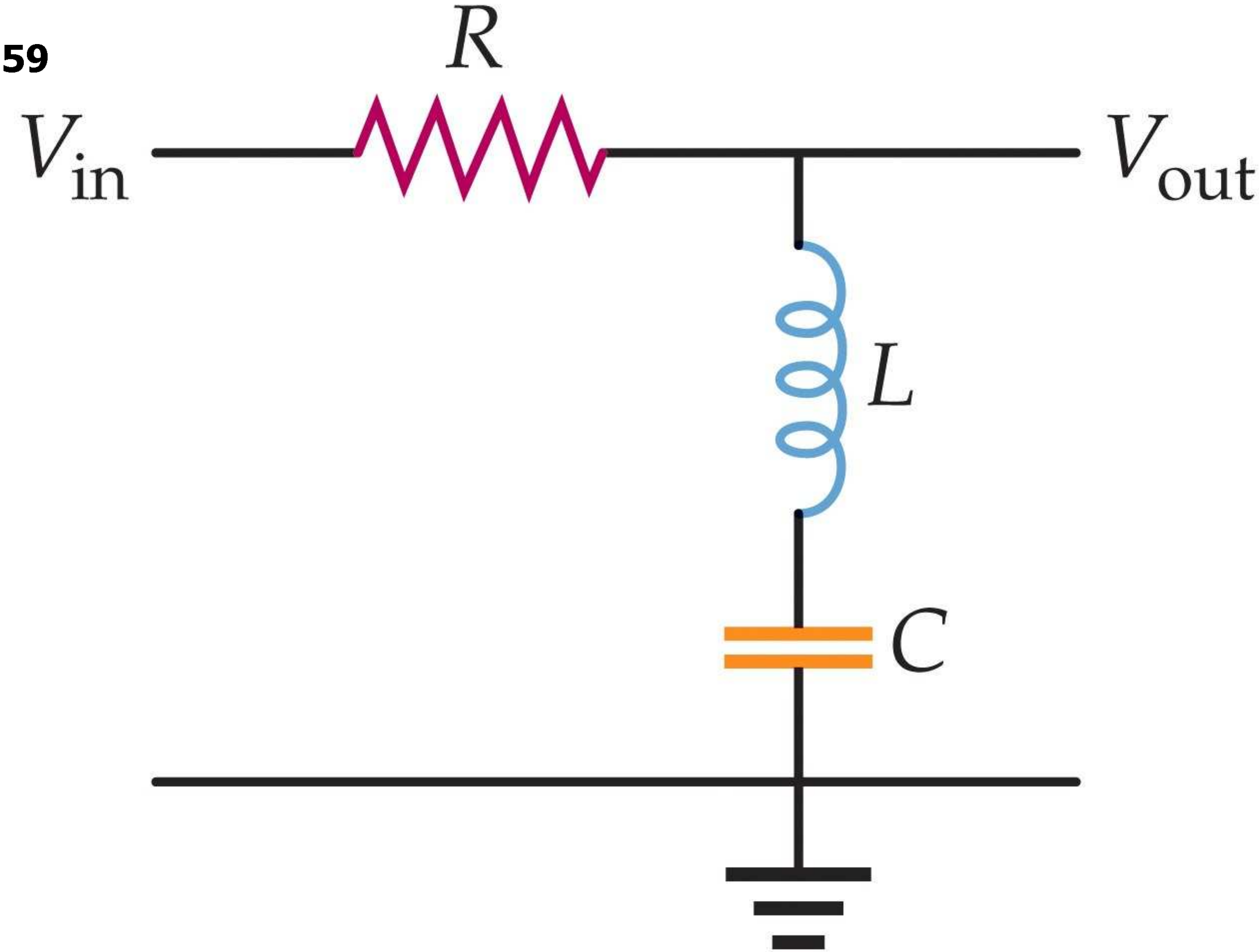

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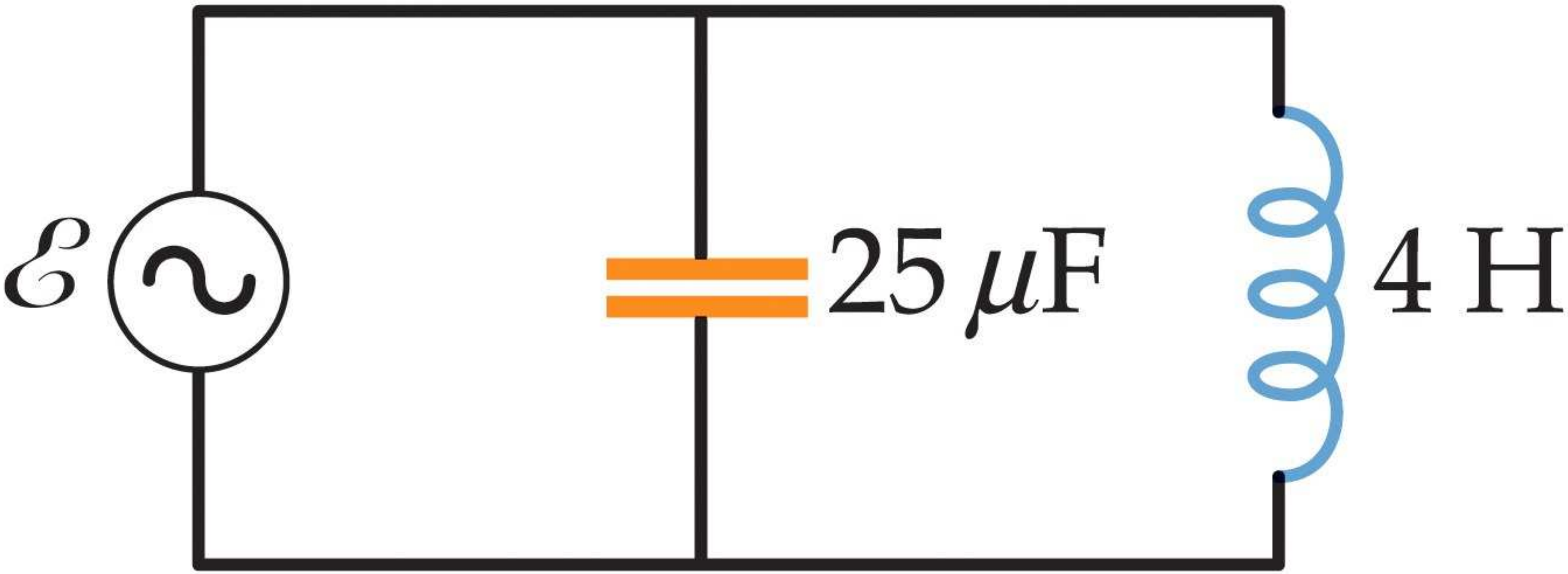

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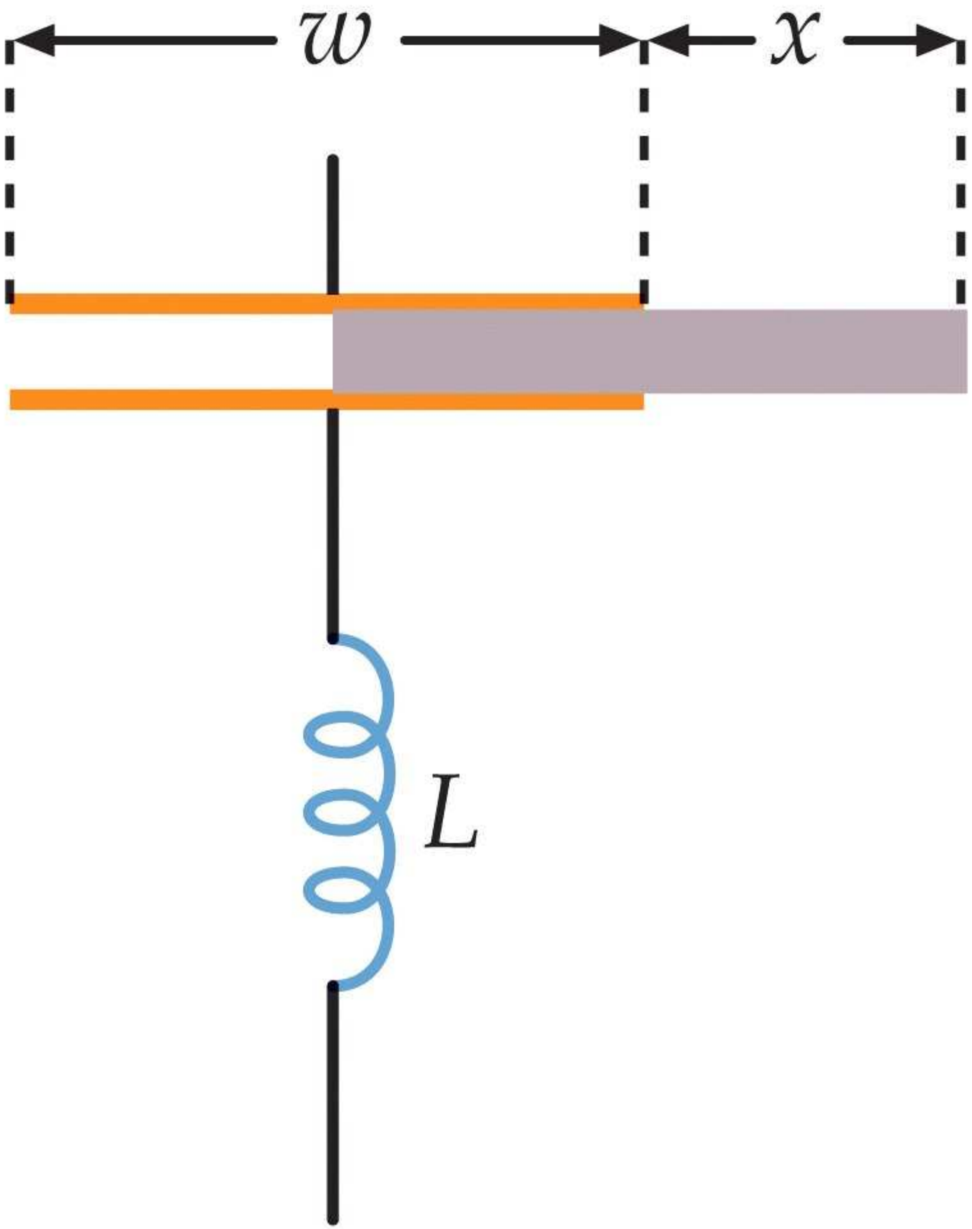

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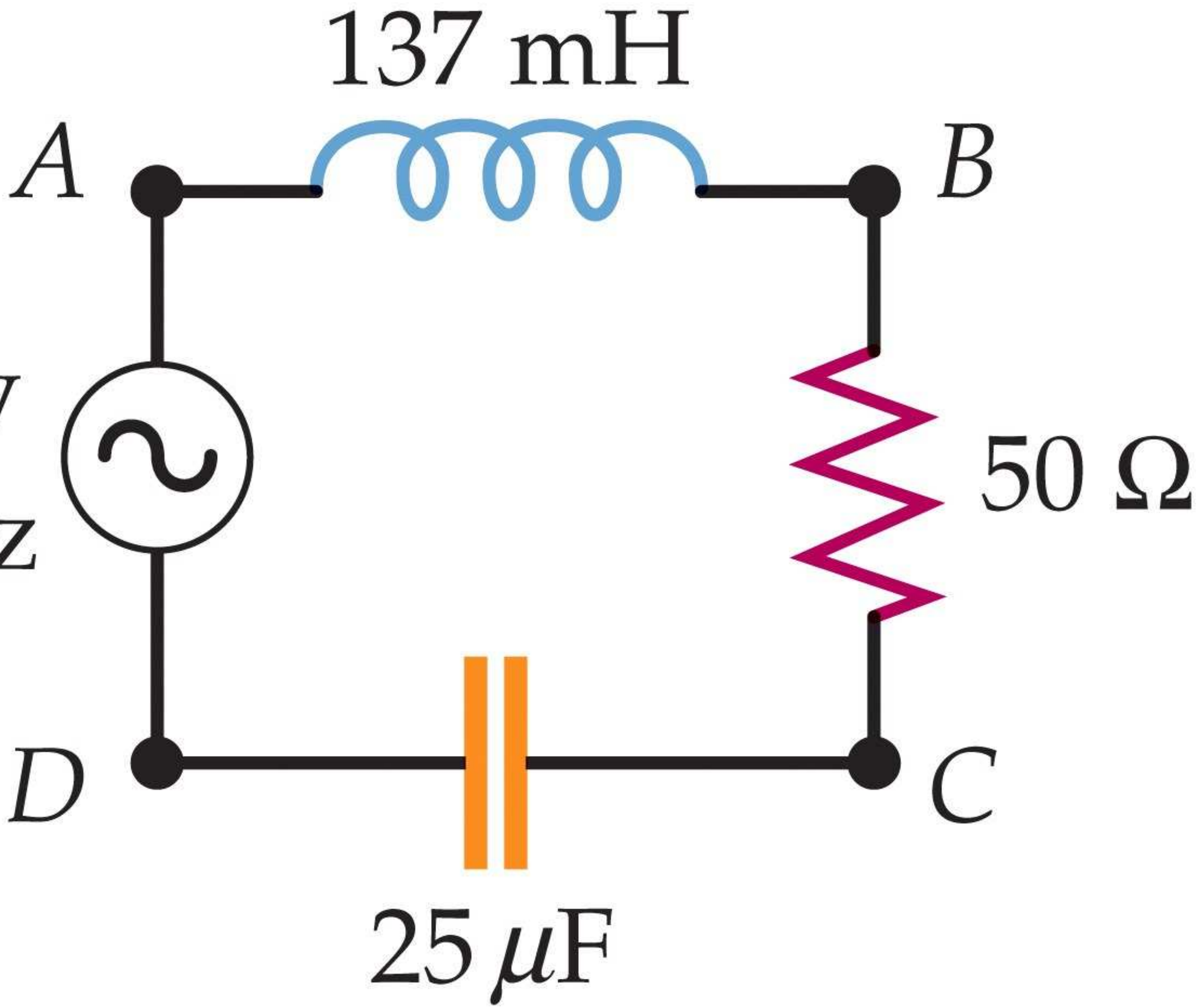

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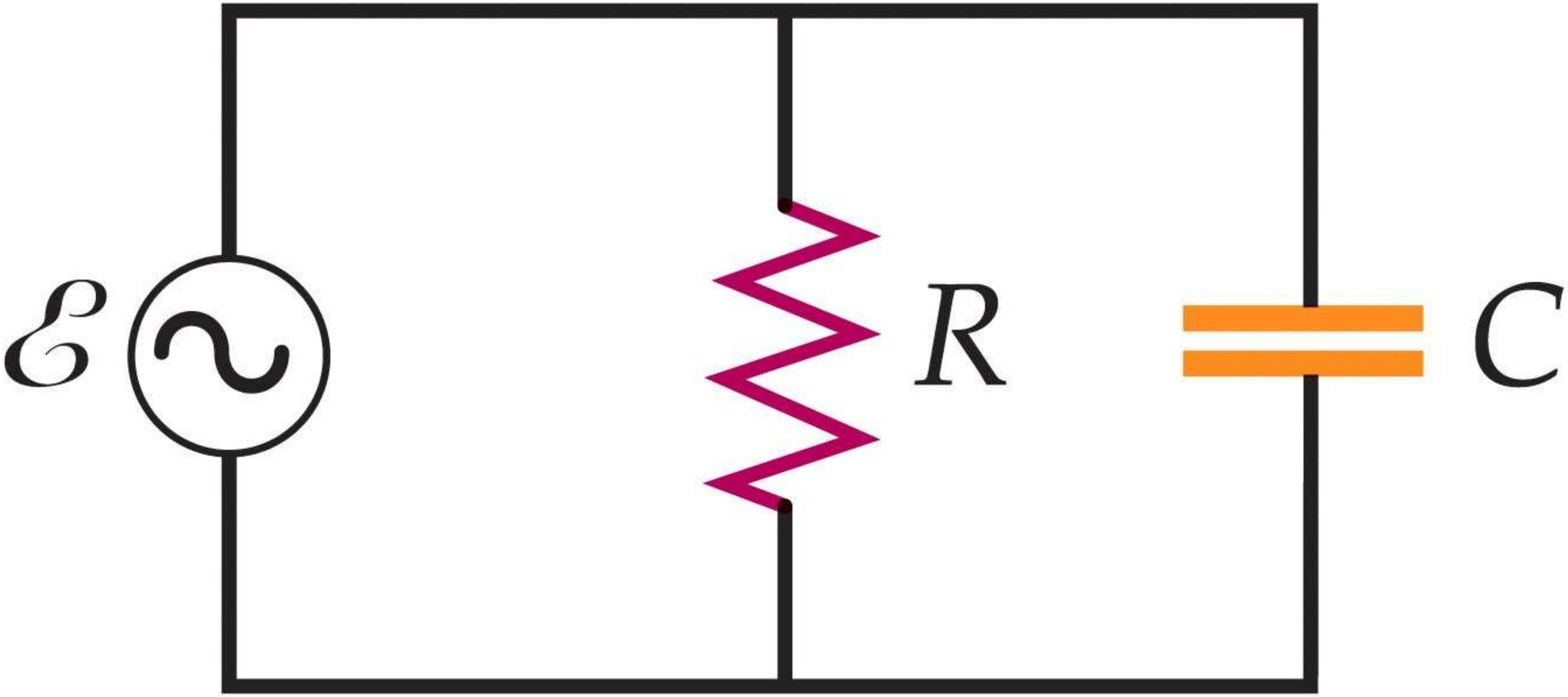

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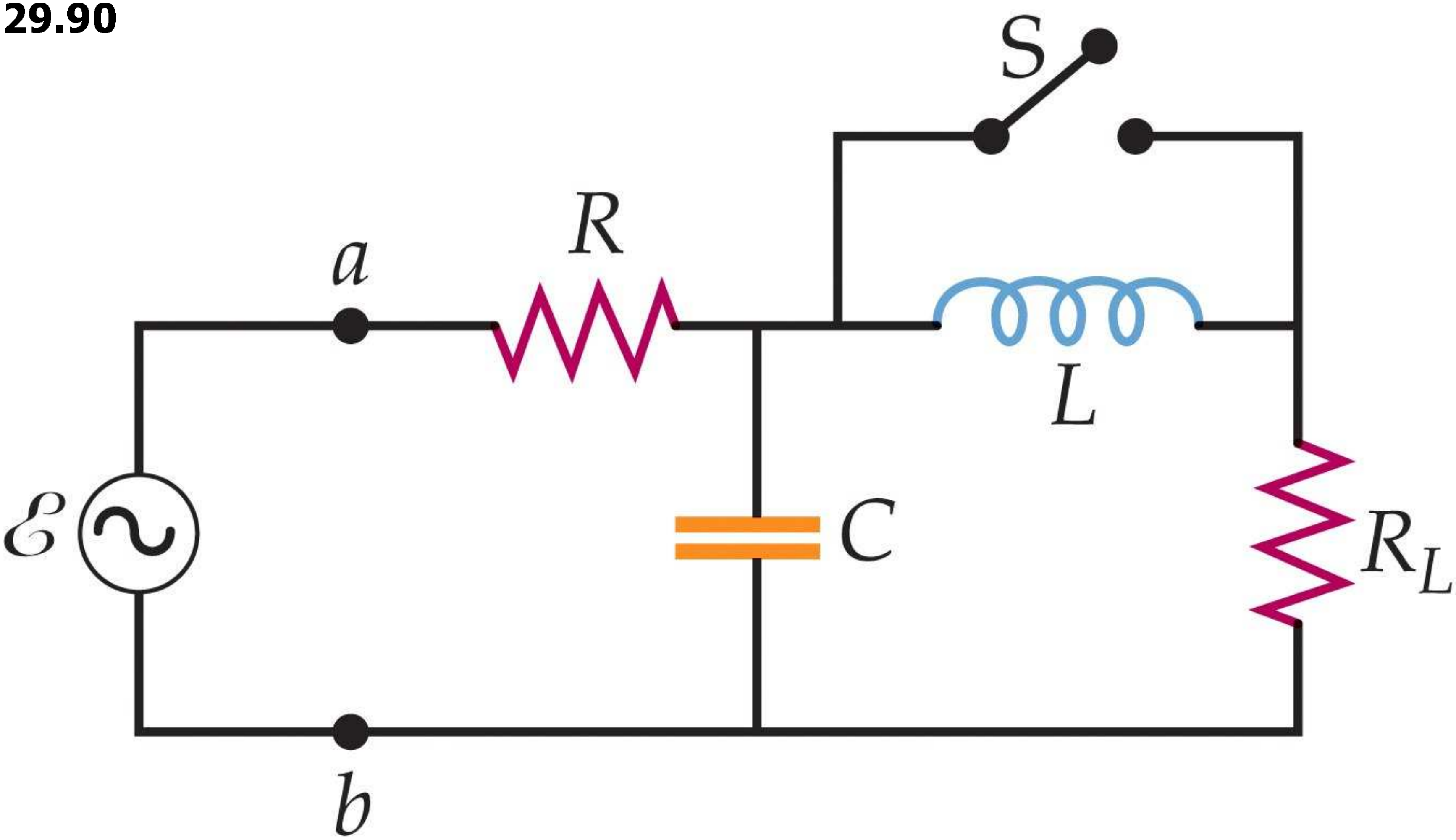

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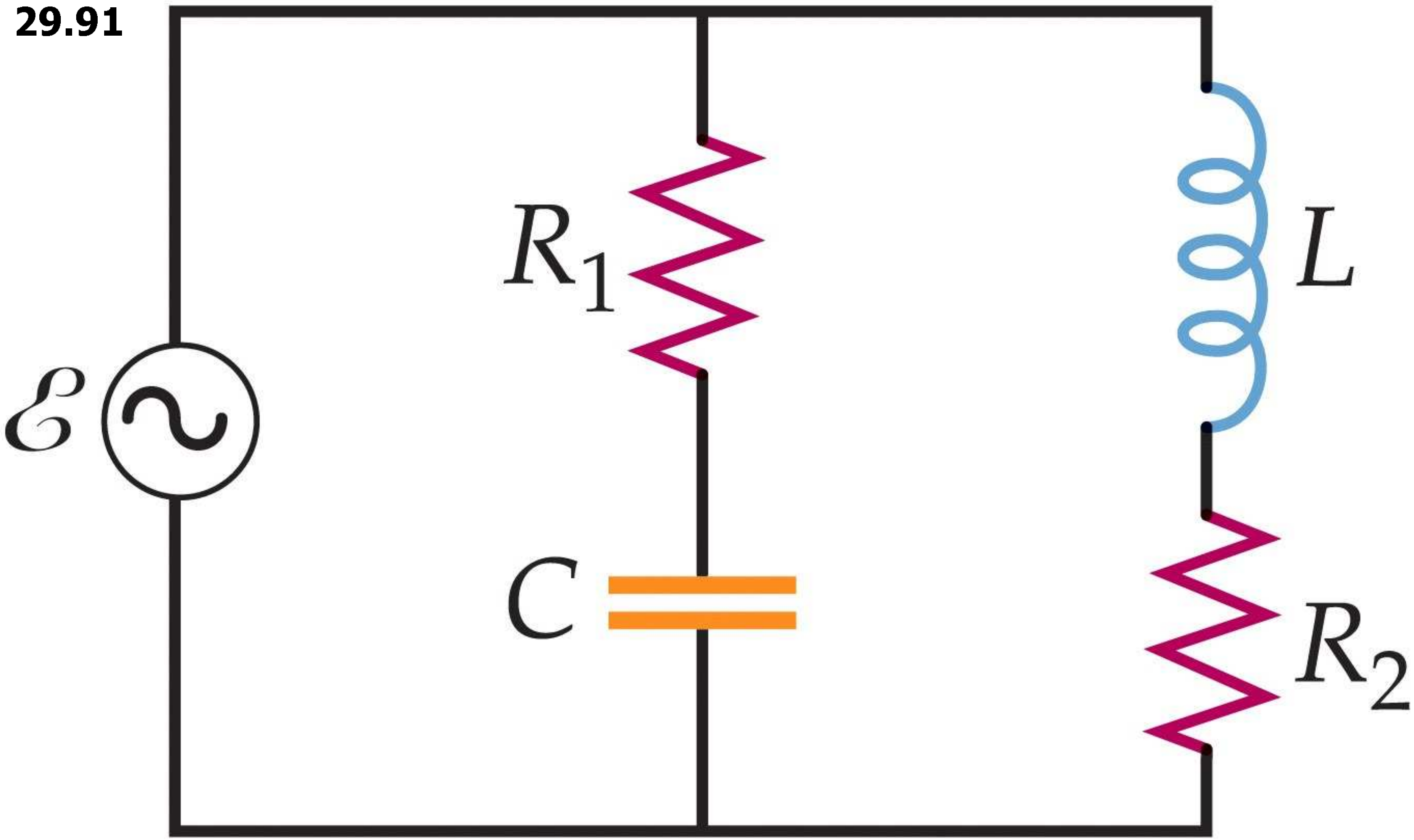

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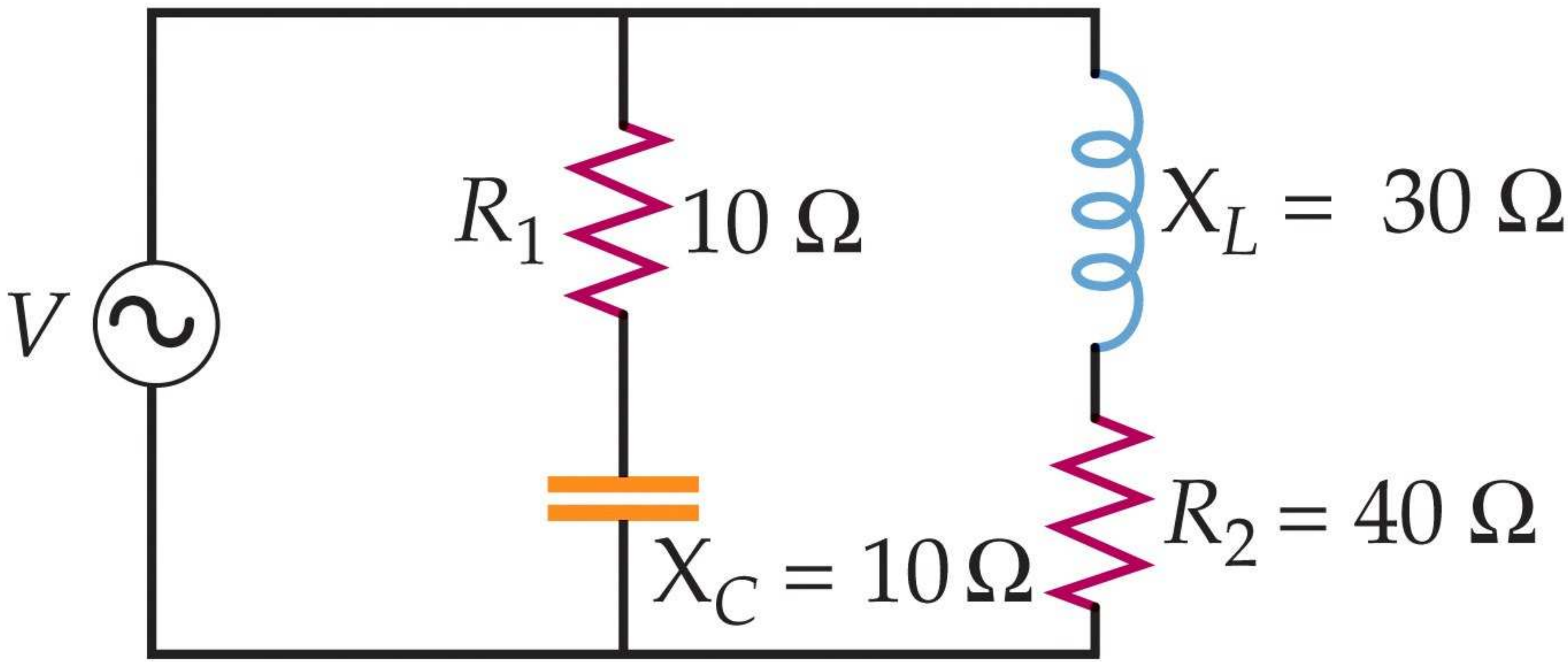

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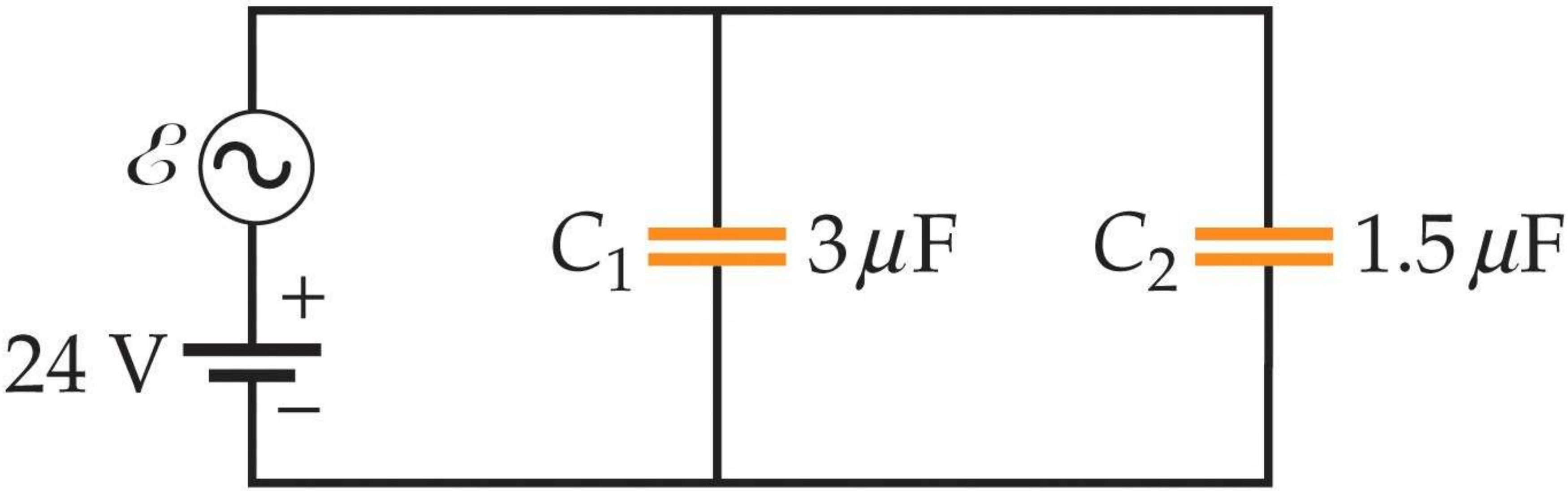

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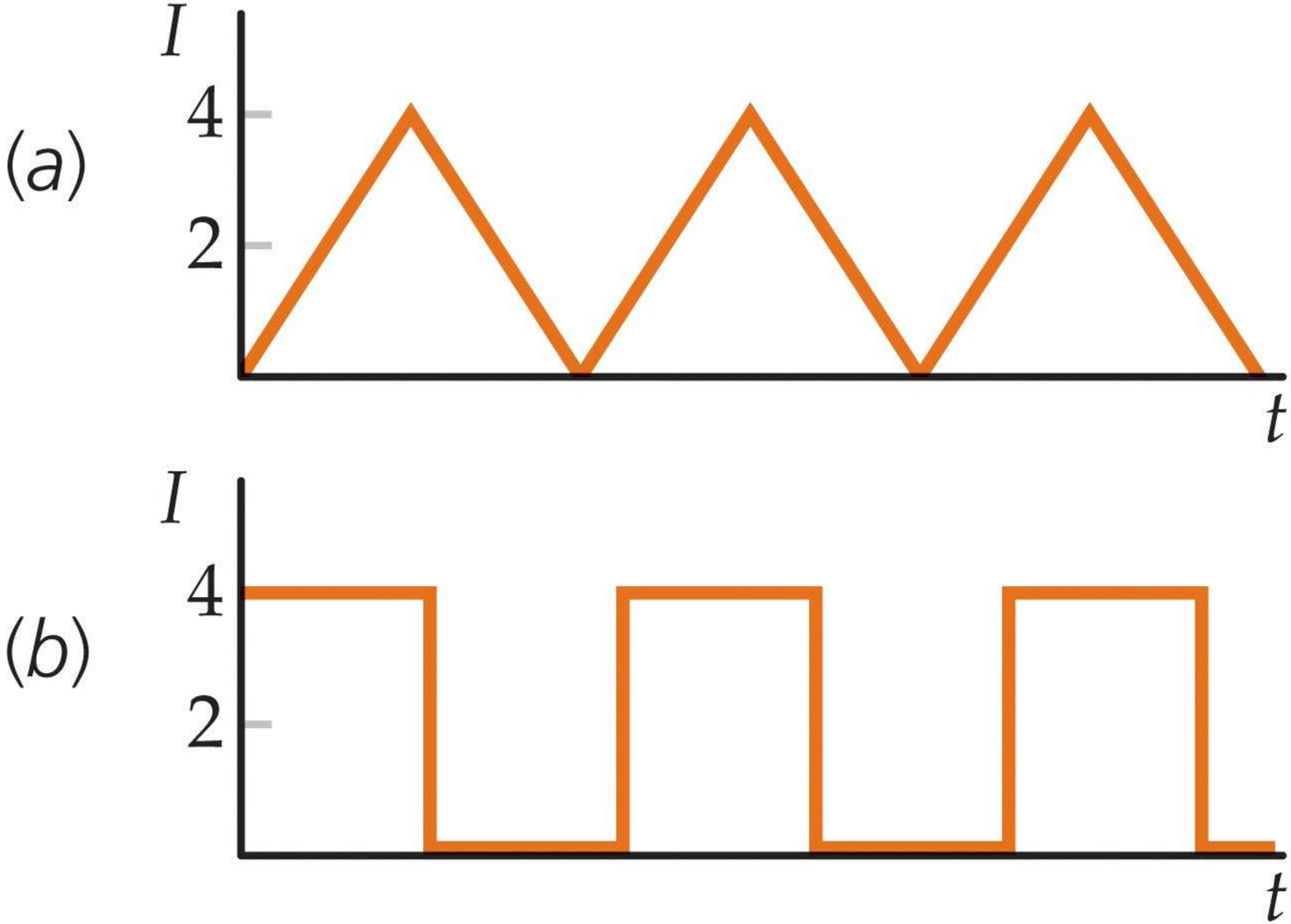

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