Capacitor C:

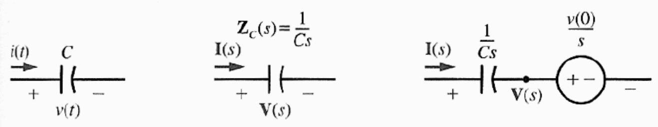
Time-domain

$$i(t) = C\frac{dv(t)}{dt}$$

$\xrightarrow{\mathcal{L}}$ Laplace or complex frequency" domain $\xleftarrow{\mathcal{L}^{-1}}$

$$\mathbf{I}(s) = C[s\mathbf{V}(s) - v(0)] = Cs\mathbf{V}(s) - Cv(0)$$

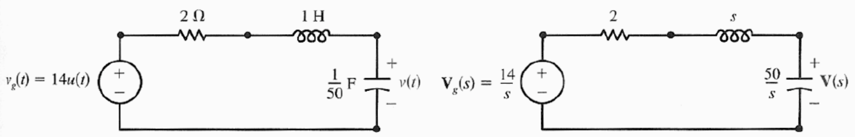
$$\mathbf{V}(s) = \frac{1}{Cs}\mathbf{I}(s) + \frac{v(0)}{s}$$



V(0) = 0

Most general case

Suppose that the series *RLC* circuit shown in Fig. 5.40*a* has zero initial conditions. Let us find the step response $v(t)$.



(a) Time domain

(b) Frequency domain

$$\longrightarrow \quad \mathbf{V}(s) = \frac{50/s}{50/s + s + 2}\mathbf{V}_g(s) = \frac{50}{s^2 + 2s + 50}\left(\frac{14}{s}\right)$$

$$V(s) = \frac{700}{s(s^2 + 2s + 50)} = \frac{K_1}{s} + \frac{K_2}{s + 1 - 7j} + \frac{K_3}{s + 1 + 7j}$$

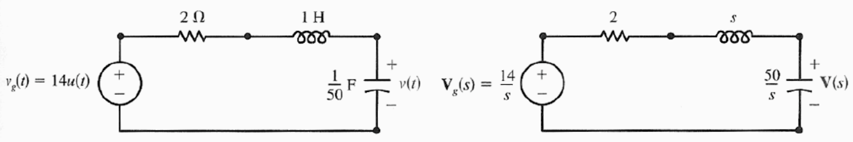
$$K_1 = \frac{700}{50} = 14 \qquad K_2 = \left.\frac{700}{s(s + 1 + 7j)}\right|_{s=-1+7j} = -7 + j$$

$$K_3 = \left.\frac{700}{s(s + 1 - 7j)}\right|_{s=-1-7j} = -7 - j = K_2^*$$

$$V(t) = 14 + 2|K_2|e^{-t}\cos(7t + \arg(K_2))$$

$$V(t) = 14 + 10\sqrt{2}e^{-t}\cos(7t + 3)$$

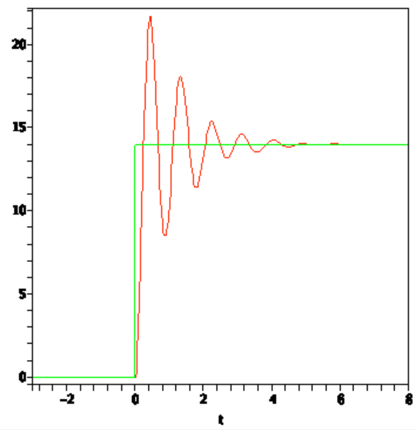
Suppose that the series *RLC* circuit shown in Fig. 5.40*a* has zero initial conditions. Let us find the step response $v(t)$.



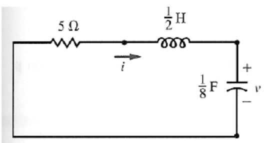
(a)

y domain

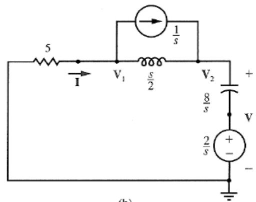
→ $\mathbf{V}(s)$



Suppose that we wish to find $v(t)$ for the series *RLC* circuit shown in Fig. 5.41*a* subject to the initial conditions $v(0) = 2$ V and $i(0) = 1$ A.



(a)



(b)

$$\frac{\mathbf{V}_1}{5} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{s/2} + \frac{1}{s} = 0 \quad \Rightarrow \quad (s + 10)\mathbf{V}_1 - 10\mathbf{V}_2 = -5$$

$$\frac{\mathbf{V}_2 - \mathbf{V}_1}{s/2} + \frac{\mathbf{V}_2 - 2/s}{8/s} = \frac{1}{s} \quad \Rightarrow \quad \text{Solve } \mathrm{V}_2.\ \text{Let } \mathrm{V}_2 = \mathrm{V}$$

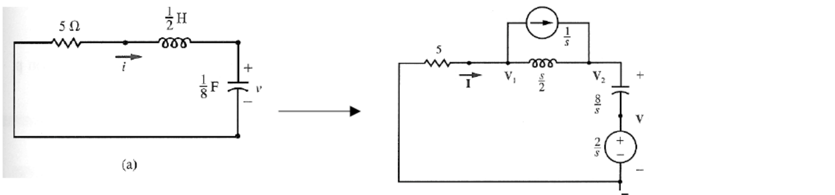
$$-16\mathbf{V}_1 + (s^2 + 16)\mathbf{V}_2 = 2s + 8$$

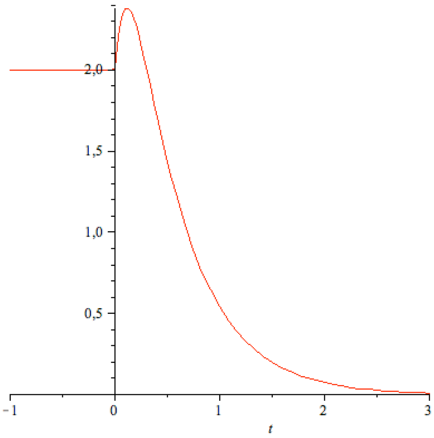
$$\mathrm{V}_2 = \frac{\begin{vmatrix} s + 10 & -5 \\ -16 & 2s + 8 \end{vmatrix}}{\begin{vmatrix} s + 10 & -10 \\ -16 & s^2 + 16 \end{vmatrix}} \longrightarrow \quad v(t) = 4e^{-2t}u(t) - 2e^{-8t}u(t)$$

$$= (4e^{-2t} - 2e^{-8t})u(t)\ \mathrm{V}$$

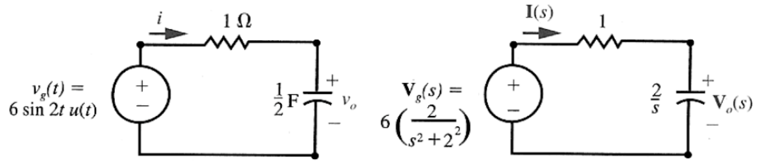
$$= \frac{(s + 10)(2s + 8) - 80}{(s + 10)(s^2 + 16) - 160} = \frac{4}{s + 2} + \frac{-2}{s + 8}$$

Suppose that we wish to find $v(t)$ for the series RLC circuit shown in Fig. 5.41a subject to the initial conditions $v(0) = 2$ V and $i(0) = 1$ A.





Let us determine the responses $i(t)$ and $v_o(t)$ to a sinusoidal excitation (which begins at time $t = 0$ s) for the circuit shown in Fig. 5.42a. The frequency-domain representation of this circuit is shown in Fig. 5.42b.



(a) Time domain

(b) Frequency domain

$$\mathbf{V}_g = 1\mathbf{I} + \frac{2}{s}\mathbf{I} = \frac{s+2}{s}\mathbf{I} \quad \Rightarrow \quad \mathbf{I} = \frac{s}{s+2}\mathbf{V}_g = \frac{12s}{(s+2)(s^2+4)}$$

$$I = \frac{K_1}{s+2} + \frac{K_2}{s-2j} + \frac{K_3}{s+2j}$$

$$K_1 = \left.\frac{12s}{s^2+4}\right|_{s=-2} = -3$$

$$K_2 = \left.\frac{12s}{(s+2)(s+2j)}\right|_{s=2j} = \frac{3}{2} - \frac{3}{2}j \quad \Rightarrow \quad \frac{3\sqrt{2}}{2}\,\angle{-\pi/4}$$

$$K_3 = K_2^*$$

$$i(t) = -3e^{-2t} + 3\sqrt{2}\cos(2t - \pi/4)$$

Next to find $v_o(t)$

$$\mathbf{V}_o = \mathbf{Z}_C\mathbf{I} = \frac{2}{s}\frac{12s}{(s+2)(s^2+4)} = \frac{24}{(s+2)(s^2+4)}$$

$$\downarrow$$

$$V_0 = \frac{K_1}{s+2} + \frac{K_2}{s-2j} + \frac{K_3}{s+2j}$$

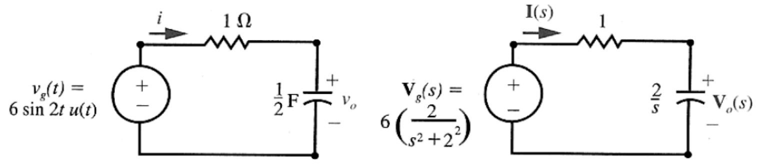
$$K_1 = \left.\frac{24}{s^2+4}\right|_{s=-2} = 3$$

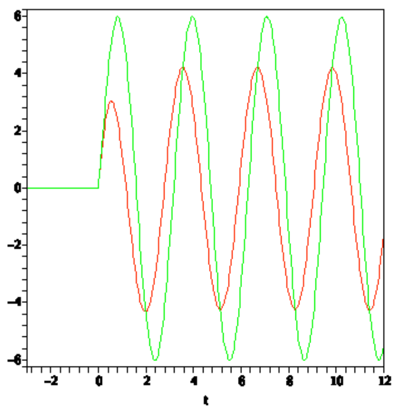
$$K_2 = \left.\frac{24}{(s+2)(s+2j)}\right|_{s=2j} = -\frac{3}{2} - \frac{3}{2}j \quad \Rightarrow \frac{3\sqrt{2}}{2}\angle{-3\pi/4}$$

$$K_3 = K_2^*$$

$$v_0(t) = 3e^{-2t} + 3\sqrt{2}\cos(2t - 3\pi/4)$$

Let us determine the responses $i(t)$ and $v_o(t)$ to a sinusoidal excitation (which begins at time $t = 0$ s) for the circuit shown in Fig. 5.42*a*. The frequency-domain representation of this circuit is shown in Fig. 5.42*b*.

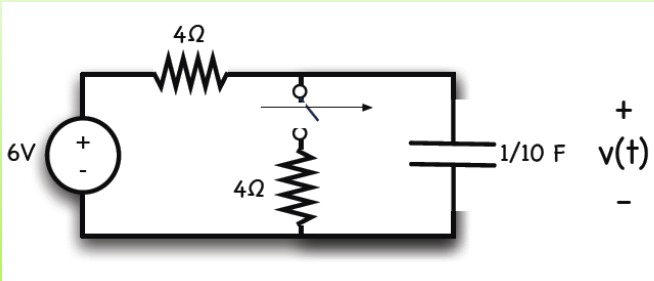


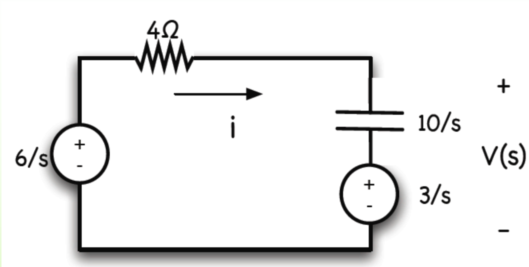


Ejemplo

- Obtener v(t)

Inicialmente v(0)=3V

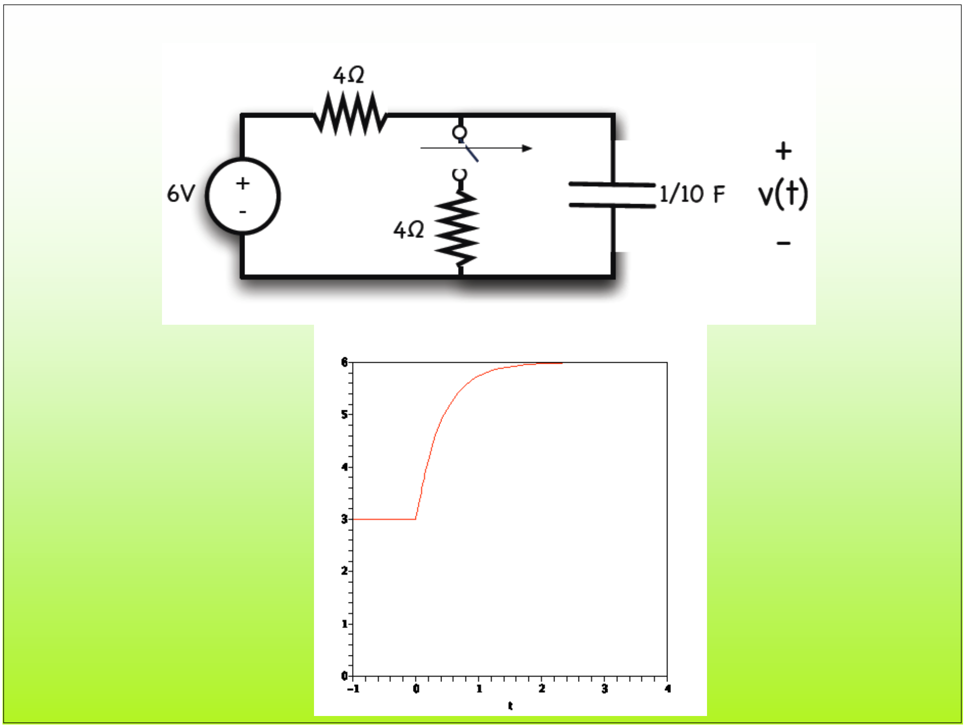




$$i = \frac{3/s}{4 + 10/s}$$

$$V(s) = \frac{3}{s} + i\frac{10}{s} = \frac{6}{s} - \frac{3}{s + 10/4}$$

$$v(t) = \left(6 - 3e^{-5t/2}\right) \ (para \ \ t > 0)$$


4Ω
6V
+
-
4Ω
1/10 F
+
v(t)
–
t