Transformada inversa

- F(s)=N(s)/D(s)
- N(s)=0 => Obtención de los ceros de la función (z_i)
- D(s)=0 => Obtención de los polos de la función (s_i)

El grado de D(s) normalmente es mayor que el grado de N(s)

Descomposición en factores simples para el caso de polos simples:

$$F(s) = \ldots + \frac{K_i}{s - s_i} + \ldots$$

$$K_i = F(s)(s - s_i)\big|_{s=p_i}$$

A systematic method to find the inverse Laplace transform

$$\mathbf{F}(s) = \frac{\mathbf{N}(s)}{\mathbf{D}(s)} = \frac{\mathbf{N}(s)}{(s - s_1)(s - s_2)(s - s_3) \cdots (s - s_n)}$$

If the degree of $\mathbf{D}(s)$ is greater than the degree of $\mathbf{N}(s)$.

$$\mathbf{F}(s) = \frac{K_1}{s - s_1} + \frac{K_2}{s - s_2} + \frac{K_3}{s - s_3} + \cdots + \frac{K_n}{s - s_n}$$

To find K_1,

$$(s - s_1)\mathbf{F}(s) = K_1 + \frac{K_2(s - s_1)}{s - s_2} + \frac{K_3(s - s_1)}{s - s_3} + \cdots + \frac{K_n(s - s_1)}{s - s_n}$$

$$\Rightarrow \quad (s - s_1)\mathbf{F}(s)\Big|_{s=s_1} = K_1$$

Find K_2, K_3, etc. similarly.

Next let us find $f_2(t)$ given that its Laplace transform is

$$\mathscr{L}[f_2(t)] = \mathbf{F}_2(s) = \frac{14s + 23}{s^2 + 5s + 4}$$

$$\mathbf{F}_2(s) = \frac{14s + 23}{s^2 + 5s + 4} = \frac{3}{s + 1} + \frac{11}{s + 4}$$

Therefore, from Table 5.1, the inverse Laplace transform of $\mathbf{F}_2(s)$ is

$$f_2(t) = 3e^{-t}u(t) + 11e^{-4t}u(t) = (3e^{-t} + 11e^{-4t})u(t)$$

$$\mathbf{F}(s) = \frac{2s^2 + 11s + 19}{(s + 1)(s + 2)(s + 3)} = \frac{K_1}{s + 1} + \frac{K_2}{s + 2} + \frac{K_3}{s + 3}$$

Multiplying this expression by $s + 1$ and then setting $s = -1$, we get

$$K_1 = \left.\frac{2s^2 + 11s + 19}{(s + 2)(s + 3)}\right|_{s=-1} = \frac{2(-1)^2 + 11(-1) + 19}{(-1 + 2)(-1 + 3)} = 5$$

Multiplying $\mathbf{F}(s)$ by $s + 2$ and then setting $s = -2$, we obtain

$$K_2 = \left.\frac{2s^2 + 11s + 19}{(s + 1)(s + 3)}\right|_{s=-2} = \frac{2(-2)^2 + 11(-2) + 19}{(-2 + 1)(-2 + 3)} = -5$$

Similarly $$K_3 = \left.\frac{2s^2 + 11s + 19}{(s + 1)(s + 2)}\right|_{s=-3} = \frac{2(-3)^2 + 11(-3) + 19}{(-3 + 1)(-3 + 2)} = 2$$

Hence $$\mathbf{F}(s) = \frac{5}{s + 1} - \frac{5}{s + 2} + \frac{2}{s + 3}$$

and from Table 5.1, the inverse Laplace transform of $\mathbf{F}(s)$ is

$$f(t) = 5e^{-t}u(t) - 5e^{-2t}u(t) + 2e^{-3t}u(t) = (5e^{-t} - 5e^{-2t} + 2e^{-3t})u(t)$$

Problema propuesto

- Realizar la antitransformada de:

$$F(s) = \frac{4s+7}{s^2+5s+4}$$

$$Sol: f(t) = 3e^{-4t} + e^{-t} \quad (t>0)$$

Caso de raíces múltiples

$$F(s) = \ldots + \frac{K_{i1}}{(s-s_i)^r} + \frac{K_{i2}}{(s-s_i)^{r-1}} + \ldots + \frac{K_{i(r-1)}}{(s-s_i)^2} + \frac{K_{ir}}{(s-s_i)} + \ldots$$

$$K_{in} = \frac{1}{(n-1)!}\frac{d^{n-1}}{ds^{n-1}}\left[F(s)(s-s_i)^r\right]\Big|_{s=s_i}$$

Ejemplo:

$$F(s)=\frac{s^2+5s+6}{s(s+1)^2} \quad \Rightarrow s_1=0, s_2=-1 \ (m_1=1, m_2=2)$$

$$F(s)=\frac{K_1}{s}+\frac{K_{21}}{(s+1)^2}+\frac{K_{22}}{s+1}$$

$$K_1=F(s)s\big|_{s=0}=6$$

$$K_{21}=\frac{1}{0!}F(s)(s+1)^2\big|_{s=-1}=-2$$

$$K_{22}=\frac{1}{1!}\frac{d}{ds}\left[\frac{s^2+5s+6}{s}\right]=-5$$

$$F(s)=\frac{6}{s}-\frac{2}{(s+1)^2}-\frac{5}{s+1}$$

$$f(t)=6-2te^{-t}-5e^{-t} \quad (t>0)$$

Caso de raices complejas

$$F(s)=......+\frac{N(s)}{(s-(\alpha+j\beta))(s-(\alpha-j\beta))}+......=.....+\frac{K}{(s-(\alpha+j\beta))}+\frac{K^*}{(s-(\alpha-j\beta))}+......$$

El valor de K se encuentra igual que en el caso de las raices simples

$$K_i=F(s)(s-s_i)\big|_{s=s_i}$$

Ejemplo

$$F(s)=\frac{5s^2+30s-15}{s\left(s^2+2s+5\right)} \quad \Rightarrow s_1=0, s_{2,3}=-1\pm 2j$$

$$F(s)=\frac{K_1}{s}+\frac{K_2}{s+1-2j}+\frac{K_3}{s+1+2j}$$

$$K_1=F(s)s\big|_{s=0}=-3$$

$$K_2=F(s)(s+1-2j)\big|_{s=-1+2j}=4-7j$$

$$K_3=K_2^*=4+7j$$

$$F(s)=-\frac{3}{s}+\frac{4-7j}{s+1-2j}+\frac{4+7j}{s+1+2j}$$

$$f(t)=-3+2\sqrt{65}e^{-t}\cos\left(2t-1.051\right) \quad (t>0)$$

Transformada inversa

$$F(s)=\frac{14s+23}{s^2+4s+5} \quad \Rightarrow s_{1,2}=-2\pm j$$

$$F(s)=\frac{K_1}{s+2-j}+\frac{K_2}{s+2+j}$$

$$K_1=F(s)(s+2-j)\big|_{s=-2+j}=7+2.5j$$

$$K_2=K_1^*=7-2.5j$$

$$F(s)=\frac{7+2.5j}{s+2-j}+\frac{7-2.5j}{s+2+j}$$

$$f(t)=\sqrt{221}e^{-2t}\cos\left(t+0.343024\right) \quad (t>0)$$

$$\frac{d^2x(t)}{dt^2} + 3\frac{dx(t)}{dt} + 2x(t) = 4e^{-3t}u(t)$$

initial conditions $x(0) = 2$ and $dx(0)/dt = -1.$

let $\mathbf{X}(s) = \mathscr{L}[x(t)],$

Taking Lapalce transform on both sides of the equation, then

$$\left[s^2\mathbf{X}(s) - sx(0) - \frac{dx(0)}{dt}\right] + 3[s\mathbf{X}(s) - x(0)] + 2\mathbf{X}(s) = \frac{4}{s+3}$$

↓ Using the initial conditions

$$s^2\mathbf{X}(s) - 2s + 1 + 3s\mathbf{X}(s) - 6 + 2\mathbf{X}(s) = \frac{4}{s+3}$$

↓

$$\mathbf{X}(s) = \frac{2s^2 + 11s + 19}{(s+1)(s+2)(s+3)}$$

↓

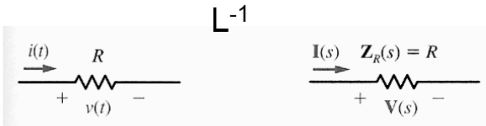
$$x(t) = (5e^{-t} - 5e^{-2t} + 2e^{-3t})u(t)$$

Application of Laplace Transform to Circuit Analysis

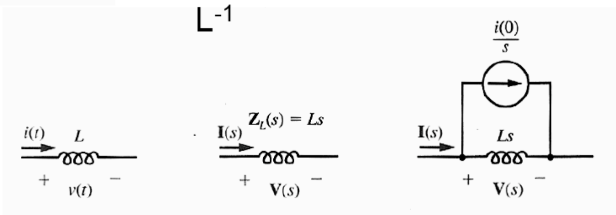
Time-domain — L → / ← L^{-1} — Laplace or complex frequency" domain

Resistor R: $v(t) = Ri(t)$ → $\mathbf{V}(s) = R\mathbf{I}(s)$

$$\mathbf{Z}_R(s) = \frac{\mathbf{V}(s)}{\mathbf{I}(s)} = R$$



Inductor L: $v(t) = L\frac{di(t)}{dt}$ → $\mathbf{V}(s) = L[s\mathbf{I}(s) - i(0)] = Ls\mathbf{I}(s) - Li(0)$



Ckt if $i(0)=0$

Most general case