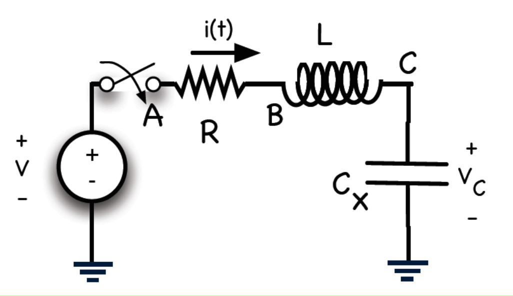
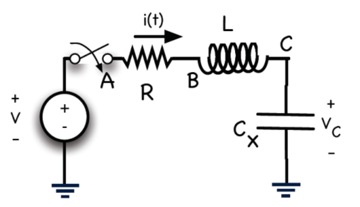
Circuito RLC



$i(t) = 0$ *Antes de cortocircuitar*

$i(t) \neq 0$ *Después de cortocircuitar*

Circuito RLC



$v_A, v_C\ ?\quad v_A(t) = u(t)V$

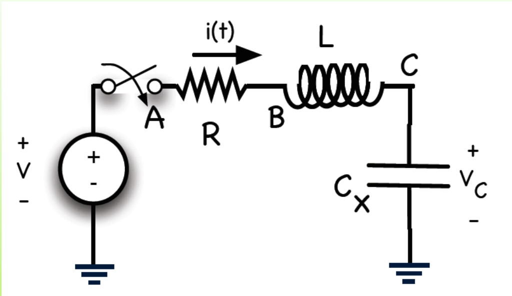
$i_R = i_L = i_C$

$$\begin{cases} \dfrac{V - v_B}{R} = i_L = i_C = C_X \dfrac{dv_C}{dt} \\ L\dfrac{di_L}{dt} = v_B - v_C \end{cases} \quad => \quad \begin{matrix} v_B = V - RC_X \dfrac{dv_C}{dt} \\ v_B = v_C + LC_X \dfrac{d^2 v_C}{dt^2} \end{matrix}$$

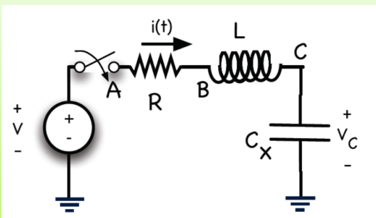
$$LC_X \frac{d^2 v_C}{dt^2} + RC_X \frac{dv_C}{dt} + v_C = V \quad <= v_C(t)$$

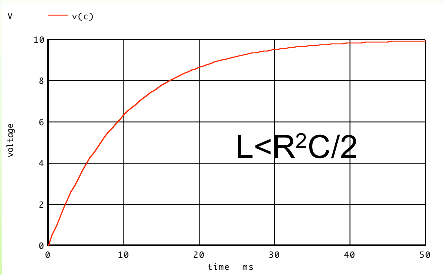
Circuito RLC

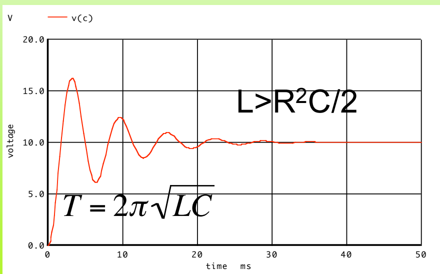
```
CIRCUIT RLC
VX A 0 DC 10
CX C 0 10u
LX B C 1m
RX A B 1K
.IC V(C)=0
.PRINT TRAN V(C)
.TRAN 0.5m 50m UIC
.END
```



Resultados de la simulación

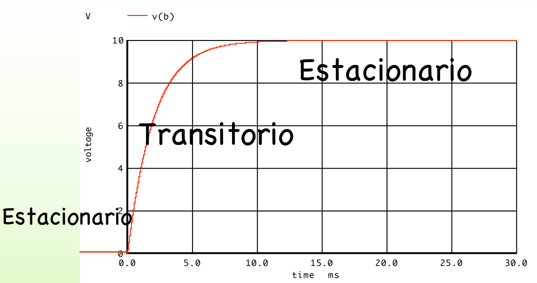


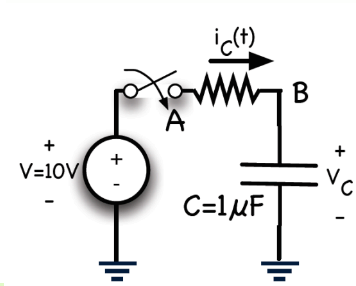


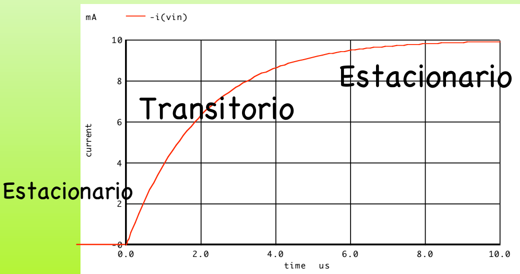


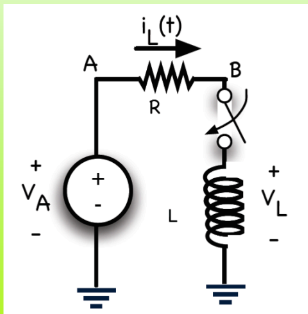
Características temporales

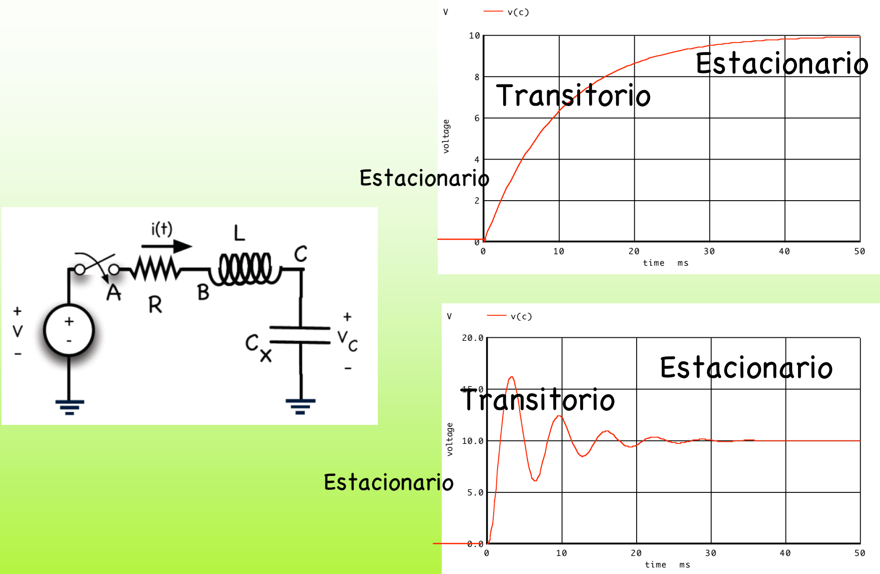
- Inicialmente las señales pasan por un período <u>transitorio</u>
- Pasado este período la señal se estabiliza y pasa a un estado <u>estacionario</u>











Uso de ecuaciones diferenciales

- Relativamente simple para circuitos RC y RL

$$RC\frac{dV_C(t)}{dt} = V - V_C(t)$$

- Complejo para circuitos con varios componentes (ej. RLC)

$$LC_X\frac{d^2v_C}{dt^2} + RC_X\frac{dv_C}{dt} + v_C = V$$

- Uso de métodologias de resolución simples: Formulismo de Laplace

La transformada de Laplace

- Simplificación de funciones temporales

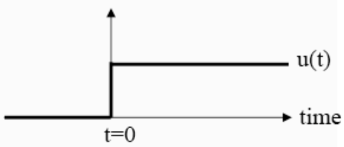
$$V(s) = \int_0^\infty v(t)e^{-st}dt$$

$$v(t) \xrightarrow{\mathcal{L}} V(s)$$

- Me permitirá usar las mismas técnicas de circuitos usadas hasta ahora

Recall the unit step function $u(t)$ defined by

$$u(t) = \begin{cases} 0 \text{ for} & t < 0 \text{ s} \\ 1 \text{ for} & t \geq 0 \text{ s} \end{cases}$$



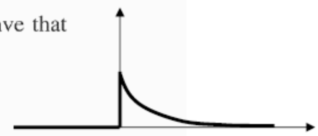
Then the Laplace transform of a unit step function is

$$\underline{\underline{\mathcal{L}[u(t)]}} = \int_0^\infty u(t)e^{-st}\,dt = \int_0^\infty e^{-st}\,dt = -\frac{1}{s}e^{-st}\Big|_0^\infty = -\frac{1}{s}(0 - 1) = \underline{\underline{\frac{1}{s}}}$$

For the decaying exponential $e^{-at}u(t)$, where $a > 0$, we have that

$$\underline{\underline{\mathcal{L}[e^{-at}u(t)]}} = \int_0^\infty e^{-at}u(t)e^{-st}\,dt = \int_0^\infty e^{-(s+a)t}\,dt$$

$$= -\frac{1}{s+a}e^{-(s+a)t}\Big|_0^\infty = -\frac{1}{s+a}(0 - 1) = \underline{\underline{\frac{1}{s+a}}}$$



Linealidad de la T.L

$\mathcal{L}[\ x(t)\ ] = X(s)$
$\mathcal{L}[\ y(t)\ ] = Y(s)$

$\mathcal{L}[\ x(t)k + y(t)r\ ] = k\ \mathcal{L}[\ x(t)\ ] + r\ \mathcal{L}[\ y(t)\ ]$
$= kX(s) + rY(s)$

Aplicación de la linealidad:

Let us find the Laplace transform of $f(t) = 3(1 - e^{-2t})u(t)$.
Since we can express this function in the form

$$f(t) = (3 - 3e^{-2t})u(t) = 3u(t) - 3e^{-2t}u(t) = f_1(t) + f_2(t)$$

where $f_1(t) = 3u(t)$ and $f_2(t) = -3e^{-2t}u(t)$. Then by the linearity property of the Laplace transform

$$\mathcal{L}[f(t)] = \mathcal{L}[3u(t)] + \mathcal{L}[-3e^{-2t}u(t)] = 3\mathcal{L}[u(t)] - 3\mathcal{L}[e^{-2t}u(t)]$$
$$= 3\left(\frac{1}{s}\right) - 3\left(\frac{1}{s+2}\right) = \frac{3(s+2) - 3s}{s(s+2)} = \frac{6}{s(s+2)}$$

Problema propuesto 3.2

- Aplicando la propiedad de linealidad. Calcular la transformada de Laplace de:

$$y(t)=2(e^{-4t}-e^{-3t})u(t)$$

$$\mathcal{L}[\ y(t)\ ] = Y(s) = ?$$

Ayuda: $\mathcal{L}[\ u(t)e^{at}\ ] = 1/(s-a)$

Tabla de Transformadas

$f(t)$	Property	$F(s)$
$f(t)$	Definition	$\int_0^\infty f(t)e^{-st}\,dt$
$f_1(t) + f_2(t)$	Linearity	$\mathbf{F}_1(s) + \mathbf{F}_2(s)$
$Kf(t)$	Linearity	$K\mathbf{F}(s)$
$\frac{df(t)}{dt}$	Differentiation	$s\mathbf{F}(s) - f(0)$
$\frac{d^2f(t)}{dt^2}$	Differentiation	$s^2\mathbf{F}(s) - sf(0) - \frac{df(0)}{dt}$
$\int_0^t f(t)\,dt$	Integration	$\frac{1}{s}\mathbf{F}(s)$
$tf(t)$	Complex differentiation	$-\frac{d\mathbf{F}(s)}{ds}$
$e^{-at}f(t)$	Complex translation	$\mathbf{F}(s+a)$
$f(t-a)\,u(t-a)$	Real translation	$e^{-as}\mathbf{F}(s)$

Note that the Laplace transform is related to the "initial conditions".

Tabla de Transformadas

f(t)	L{f(t)}	f(t)	L{f(t)}
$\delta(t)$	1	u(t)	$\frac{1}{s};\ s>0$
e^{at}	$\frac{1}{s-a};\ s>a$	t	$\frac{1}{s^2};\ s>0$
$\sin(at)$	$\frac{a}{s^2+a^2};\ s>0$	$\cos(at)$	$\frac{s}{s^2+a^2};\ s>0$
$t^n\ e^{at}$	$\frac{n!}{(s-a)^{n+1}};\ s>a$	$e^{at}\ \sin(kt)$	$\frac{k}{(s-a)^2+k^2};\ s>a$
$e^{at}\ \cos(kt)$	$\frac{s-a}{(s-a)^2+k^2};\ s>a$	$t^n;\ \ n=\{1,2,...\}$	$\frac{n!}{s^{n+1}};\ \ s>0$
$2\lvert K\rvert e^{\alpha t}\cos(\beta t+\arg K)$	$\frac{K}{s-\alpha-j\beta}+\frac{K^*}{s-\alpha+j\beta}$		
$cf(t)$	cF(s)		
$f_1(t)+f_2(t)$	$F_1(s)+F_2(s)$		
$f'(t)$	$sF(s)-f(0)$		
$f^{(n)}(t)$	$s^nF(s)-s^{n-1}f(0)-...-f^{(n-1)}(0)$		
$t^n\ f(t)$	$(-1)^n\ F^{(n)}(s)$		
$e^{at}\ f(t)$	$F(s-a)$		
$\delta(t-a)$	e^{-as}		

Ventajas de la T.L.

- Las ecuaciones diferenciales se convierten en ecuaciones algebraicas

$$RC\frac{dV_C(t)}{dt}=V-V_C(t)$$

$$V_C(0)=\frac{dV_C(0)}{dt}=0\quad (Condiciones\ \ iniciales)$$

$$\downarrow$$

$$RC\left[sV_C(s)-V_C(0)\right]=\frac{V}{s}-V_C(s)$$

$$V_C(s)=\frac{V}{s(1+sRC)}\xrightarrow{\mathcal{L}^{-1}}V_C(t)=V\left(1-e^{-\frac{t}{RC}}\right)u(t)$$