Three-terminal transport through a quantum dot in the Kondo regime:
Conductance, dephasing, and current-current correlations

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We investigate the nonequilibrium transport properties of a three-terminal quantum dot in the strongly interacting limit. At low temperatures, a Kondo resonance arises from the antiferromagnetic coupling between the localized electron in the quantum dot and the conduction electrons in source and drain leads. It is known that the local density of states is accessible through the differential conductance measured at the (weakly coupled) third lead. Here, we consider the multiterminal current-current correlations (shot noise and cross correlations measured at two different terminals). We discuss the dependence of the current correlations on a number of external parameters: bias voltage, magnetic field, and magnetization of the leads. When the Kondo resonance is split by fixing the voltage bias between two leads, the shot noise shows a nontrivial dependence on the voltage applied to the third lead. We show that the cross correlations of the current are more sensitive than the conductance to the appearance of an external magnetic field. When the leads are ferromagnetic and their magnetizations point along opposite directions, we find a reduction of the cross correlations. Moreover, we report on the effect of dephasing in the Kondo state for a two-terminal geometry when the third lead plays the role of a fictitious voltage probe.

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I. INTRODUCTION

The Kondo effect represents a distinguished example of strong many-body correlations in condensed matter physics. Over the last 15 years, much effort has been made in understanding the implications of the Kondo effect on the scattering properties of phase-coherent conductors. Indeed, the electric transport through a quantum dot connected to two terminals becomes highly correlated when the temperature $T$ is lowered below a characteristic energy scale given by $k_B T_K$. At equilibrium, the Kondo temperature $T_K$ depends on the parameters of the system, i.e., the coupling of the dot to the external leads due to tunneling, the dot onsite repulsion (charging energy) and the position of the resonant level relative to the Fermi energy $E_F$. All of them can be tuned in a controlled manner.

In a quantum dot with a sufficiently large charging energy ($U \gg k_B T$) and a single energy level well below $E_F$, the dynamics of the quasilocalized electron becomes almost frozen. Therefore, a quantum dot can be viewed as an artificial real-localization. This issue has recently attracted a lot of attention. In this work, we mimic, in a phenomenological way, the effect of dephasing on the transport properties of a three-terminal quantum dot in the Kondo regime by introducing a fictitious voltage probe.

Now, in the absence of dephasing, the building block of the Kondo resonance is a narrow peak (of width $\sim k_B T_K$) around $E_F$ in the local density of states (LDOS) of the dot. However, full quantum-dot spectroscopy of the LDOS cannot be accomplished with a two-terminal transport setup. In particular, one cannot gain experimental access to the predicted large voltage induced splitting of the LDOS when $eV_{sd} > k_B T_K$. A way to circumvent this problem is by attaching a third lead, as demonstrated independently by Sun and Guo and Lebanon and Schiller. In subsequent laboratory work, De Franceschi et al. observed a split Kondo resonance by employing a slightly modified technique—one of the leads was replaced by a narrow wire driven out of equilibrium where left and right moving carriers have different electrochemical potentials.

Motivated in part by the works cited in the preceding paragraph, we are concerned in this paper as well with the nonequilibrium Kondo physics and the fluctuations of the current through a quantum dot attached to three leads. As is well known, the investigation of the current-current correlations in mesoscopic conductors has been a fruitful area of research. Nevertheless, there are still very scarce applications to strongly correlated systems as the shot noise is a purely nonequilibrium property, and thus more difficult to treat. Hershfield calculates the zero-frequency shot noise using perturbation theory in the charging energy (valid when the Kondo correlations are not large; e.g., at $T > T_K$). Yamaguchi and Kawamura choose the tunneling part of the Hamiltonian as the perturbing parameter. Ding and Ng study the frequency dependence of the noise by means of the equation-of-motion method (also reliable for $T > T_K$). Meir...
and Golub perform an exhaustive study of the influence of bias voltage in the shot noise of a quantum dot in the Kondo regime. Dong and Lei discuss the effect on the shot noise of both external magnetic fields and particle-hole symmetry breaking. Avishai et al. calculate the Fano factor when the leads are s-wave superconductors whereas the case of p-wave superconductivity is treated by Aono et al. The authors examine the behavior of the Fano factor at zero temperature when the formation of the Kondo resonance competes with the presence of ferromagnetic leads and spin-flip processes. López et al. make use of the two-impurity Anderson Hamiltonian to address the shot noise in double quantum dot systems. To the best of our knowledge, a study of the current fluctuations in a multiprobe Kondo impurity is still missing. This is the gap we want to fill here.

In mesoscopic conductors, Büttiker shows that the sign of the current cross correlations depends on the statistics of the carriers. It is positive (negative) for bosons (fermions) due to statistical bunching (antibunching). This statement is based on a series of assumptions (e.g., zero-impedance external circuits, spin-independent transport, normal thermal leads). Positive correlations can be found if these conditions are not met (see Ref. 25 for references on this subject). Here, we just mention a few studies based on structures involving a quantum dot. Bagrets and Nazarov consider a Coulomb-blockaded quantum dot attached to paramagnetic leads whereas the ferromagnetic case and the spin-blockade case are treated by Cottet et al. Börlin et al. and Samuelsson and Büttiker examine the cross correlations of a chaotic dot in the presence of a superconducting lead. In the spin-dependent case, Sánchez et al. find that the sign of the cross correlations is affected by Andreev cross reflections. In the context of quantum computation, measuring current cross correlations have been shown to yield a indirect identification of the existence of streams of entangled particles. Therefore, the cross correlations are a valuable tool in characterizing the electron transport in phase-coherent conductors.

In this work, we consider electron transport through a strongly interacting quantum dot attached to three leads (see Fig. 1). Section II explains the theoretical framework (slave-boson mean-field theory) we use to compute the conductance and the current-current correlations. We show that the expressions for the cross correlations may be inferred from scattering theory applied to a Breit-Wigner resonance with renormalized parameters. Section III is devoted to the results. First, we assume that the third lead is a fictitious voltage probe and investigate the effect of dephasing with increasing coupling to the probe. Then, we consider that lead as a real electrode and relate the differential conductance measured at one electrode with the local density of states (LDOS) of the artificial Kondo impurity. We show next that the sign of the cross correlations of the current is negative, as expected from the fermionic character of the Kondo correlations at very low temperature. Moreover, we discuss the effect of bias voltage, external magnetic fields, and spin-polarized tunneling in the cross correlations. We finish this section with an investigation of the effect of spin polarized transport in the shot noise. Finally, Sec. IV contains our conclusions.

II. MODEL

We model the electric transport through the quantum dot using the Anderson Hamiltonian in the limit of large onsite Coulomb interaction $U \rightarrow \infty$. This way we neglect double occupancy in the dot and the Hamiltonian is written in terms of the slave-boson language

$$\mathcal{H} = \sum_{k\sigma} e_{k\sigma} c_{k\sigma}^\dagger c_{k\sigma} + \sum_{\alpha} n_{\alpha} \epsilon_{\alpha} + \sum_{k\sigma} (V_{k\sigma} c_{k\sigma}^\dagger b_{\sigma} + H.c.) + \lambda \left( b^\dagger b + \sum_{\sigma} f_{\sigma}^\dagger f_{\sigma} - 1 \right),$$

(1)

where $c_{k\sigma}^\dagger$ ($c_{k\sigma}$) is the creation (annihilation) operator describing an electronic state $k$ with spin $\sigma=\uparrow, \downarrow$ and energy dispersion $\epsilon_{k\sigma}$ in the lead $\alpha=1,2,3$. $n_{\alpha}$ is the (spin-dependent) energy level in the dot and $V_{k\sigma}$ is the coupling matrix element. The original dot second-quantization operators have been replaced in Eq. (1) by a combination of the pseudofermion operator $f_{\sigma}$ and the boson field $b$. Hopping off the dot is described by the process $c_{k\sigma}^\dagger b_{\sigma}$, whenever an electron is annihilated by $c_{k\sigma}^\dagger$, an empty state in the dot is created by $b_{\sigma}$, and then $b_{\sigma}$ generates an electron with spin $\sigma$ in the conduction band of contact $\alpha$. The boson operator $b$ ($b^\dagger$) may be regarded as a projection operator onto the vacuum (empty) state of the quantum dot. To make sure that a state with double occupancy is never created, a constraint with Lagrange multiplier $\lambda$ is added to the Hamiltonian.

The current operator $\hat{I}_\alpha$ that yields the electronic flow from lead $\alpha$ is given by

$$\hat{I}_\alpha = \frac{i e}{h} [\hat{N}_\alpha, \mathcal{H}],$$

(2)

where $\hat{N}_\alpha = \sum_{k\sigma} c_{k\sigma}^\dagger c_{k\sigma}$. The general form of the power spectrum of the current fluctuations reads

$$S_{\alpha\beta}(\omega) = 2 \int d\tau e^{i\omega \tau} \langle \{ \hat{I}_\alpha(\tau), \hat{I}_\beta(0) \} \rangle$$

$$= 2 \int d\tau e^{i\omega \tau} \langle \{ \langle \hat{I}_\alpha(\tau), \hat{I}_\beta(0) \rangle \rangle - \langle \hat{I}_\alpha \rangle \langle \hat{I}_\beta \rangle \} \rangle,$$

(3)

$\delta I_\alpha = \hat{I}_\alpha - \langle I_\alpha \rangle$ describing the fluctuations of the current away from the mean value.
from its average value \( I_a = \langle \hat{I}_a \rangle \). We are interested in the zero-frequency limit of \( S_{ab}(\omega) \). Since the energy scale \( k_B T_K \) in typical experiments is of the order of 100 mK, the frequencies should be \( \omega \approx 2.4 \) GHz. Moreover, we shall work at \( T = 0 \) (see below) so that the current will fluctuate due to quantum fluctuations only (we disregard thermal fluctuations).

A. Mean-field approximation

The mean-field solution of the Hamiltonian (1) consists of considering the effect of the boson in an averaged way, replacing the true operator \( \hat{b}(t) \) by its expectation value \( \langle \hat{b}(t) \rangle \). Within this approximation the Hamiltonian describes noninteracting quasiparticles with renormalized couplings: 
\[
V_{k \alpha}^{G \prime} \rightarrow \tilde{V}_{k \alpha}^{G \prime}
\]
The theory is then suitable for studying the Fermi-liquid fixed point of the Kondo problem (i.e., at \( T = 0 \)) in which the averaged occupation in the dot is always \( 1 \). The dominant fluctuations in the system are those associated to spin.

The stationary state of the boson field is determined from the \( t \to \infty \) limit of its equation of motion using the Keldysh contour. Therefore, we obtain a closed system of two nonlinear equations [Eqs. (4) and (5)] with unknowns \( |b|^2 \) and \( \lambda \) to be found self-consistently.

From the precedent arguments and Eq. (2) we can easily establish an expression for the expectation value of the electric current
\[
I_a = \frac{e}{\hbar} \sum_{\mu \alpha} \int d\epsilon \tilde{T}^{\alpha}_{ab}(\epsilon)[f_{\alpha}(\epsilon) - f_{\beta}(\epsilon)],
\]
which has exactly the same transparent form as the Landauer-Büttiker formula \(^{39} \) in the two channel (one per spin) case applied to a double-barrier resonant-tunneling system
\[
\tilde{T}^{\alpha}_{ab}(\epsilon) = -\frac{4\Gamma_{ab}^{\alpha} \tilde{G}^{\alpha}_{ab}}{(\epsilon - \epsilon_{0\alpha})^2 + \Gamma^2_{ab}},
\]
which has a simple Breit-Wigner line shape. For the same reason, the quasiparticle density of states is a Lorentzian function centered around the Fermi level (the Abrikosov-Suhl resonance). This result is expected since we are dealing with a Fermi liquid, but we stress that the physics it contains should not be confused with a noninteracting quantum dot because:

(i) \( \tilde{T} \) depends implicitly on \( |b|^2 \) and \( \lambda \), and it must then be self-consistently calculated for each set of parameters: contact voltages \( \{V_g\} \), magnetic field \( \Delta \approx \epsilon_{0\uparrow} - \epsilon_{0\downarrow} \), gate voltage \( \epsilon_{0\alpha}(V_g) \), and lead magnetization.

(ii) \( \tilde{T} \) is renormalized by Kondo correlations (as the bare \( \Gamma \) and \( \epsilon_{0\alpha} \) are).

(iii) \( \tilde{T} \) has a nontrivial dependence on the bias voltage.

All these features give rise to a number of effects that are not seen in a noninteracting resonant-tunneling diode. There are many instances: regions of negative differential conductance in the current-voltage characteristics of a double quantum dot, \(^{37} \) a crossover from Kondo physics to an antiferromagnetic singlet in the two-impurity problem, \(^{23} \) an anomalous sign of the zero-bias magnetoresistance, \(^{22} \) etc. Below, we discuss another example without counterpart in a noninteracting Breit-Wigner resonance: When the Kondo peak splits due to a large bias voltage.

B. Current-current correlations

We consider now the current fluctuations given by Eq. (3) at zero frequency \( S_{ab}(0) \). To simplify the notation we introduce \( G_0(\omega) = G_{f \alpha \beta}^{\alpha \beta}(\omega) \) as the dot Green function. After lengthy algebra, we have
\[
S_{ab}(0) = \frac{4e^2}{\hbar} \int d\epsilon \tilde{T}^{\alpha}_{ab} \left[ G^\alpha_{ab} G^\alpha_{ab} - G^\alpha_{ab} G^\alpha_{ab} f_a + G^\alpha_{ab} G^\alpha_{ab} (1 - f_b) \right]
\]
\[
- G^\alpha_{ab} G^\alpha_{ab} f_a + G^\alpha_{ab} G^\alpha_{ab} f_b - G^\alpha_{ab} G^\alpha_{ab} f_a f_b(1 - f_b)
\]
\[
- G^\alpha_{ab} G^\alpha_{ab} f_b(1 - f_a) - i \frac{\delta_{\alpha \beta}}{\pi \tilde{T}^{\alpha}_{ab}} (G^\alpha_{ab} (1 - f_b) - G^\alpha_{ab} f_a) \right].
\]

This formula (or variations of it) has been already employed in the literature. Wei et al. \(^{38} \) prove it using the Fisher-Lee-Baranger-Stone relation \(^{39} \) to write the scattering matrix.
elements in terms of the retarded Green function of the dot, $G'_0$. Dong and Lei,\textsuperscript{19} and López \textit{et al.}\textsuperscript{23} find it in Kondo problems within a slave-boson mean-field framework. Actually, in Ref. 23 it is shown that the shot noise in a two-terminal geometry reads $S \sim T(1-T)$, i.e., the well-known result for the partition noise, but with \textit{renormalized} transmissions. Souza \textit{et al.}\textsuperscript{40} calculate the noise of an ultrasmall magnetic tunnel junction by means of Eq. (9) within a Hartree-Fock framework. In general, we can say that Eq. (9) is consistent within mean-field theories. However, some caution is needed if one wishes to go beyond a mean-field level. In deriving Eq. (9), one needs to apply Wick theorem, which is valid only for \textit{noninteracting} (quasi)particles. More specifically, one finds terms that read

\begin{equation}
\langle c_{ka\sigma}^\dagger(t) f_{\sigma}(t) c_{b\bar{\sigma}}(0) f_{\bar{\sigma}}(0) \rangle = \langle c_{ka\sigma}^\dagger(t) \rangle \langle f_{\sigma}(t) \rangle \langle c_{b\bar{\sigma}}^\dagger(0) \rangle + \langle c_{ka\sigma}^\dagger(t) \rangle \langle f_{\bar{\sigma}}(0) \rangle \\
\times \langle f_{\sigma}(t) \rangle \langle c_{b\bar{\sigma}}^\dagger(0) \rangle.
\end{equation}

The first term in the left-hand side corresponds to disconnected diagrams that cancel out the term $\langle \hat{I}_\alpha \rangle \langle \hat{I}_\beta \rangle$ of Eq. (3) whereas the second term contributes to Eq. (9). Therefore, the particular Hamiltonian has to be cast first in a quadratic form. Zhu and Balatsky\textsuperscript{41} incorrectly state that Eq. (9) takes into account the many-body effects. Also, it is not clear how this formula is inferred within the \textit{equation-of-motion} method employed by Lü and Liu.\textsuperscript{42}

In our case, the mean-field approximation is known to be the leading term in a $1/N$ expansion,\textsuperscript{43} where $N=2$ is the spin degeneracy. Therefore, we neglect the fluctuations of both the boson field ($\partial_0=0$) and the renormalization of the resonant level ($\partial_0=0$),\textsuperscript{19,32} which could be calculated in the next order. This is valid as long as we restrict ourselves to the Fermi-liquid fixed point of the Kondo problem. We are not aware of real $1/N$ correction calculations of shot noise. Although Meir and Golub\textsuperscript{18} perform a noncrossing approximation (NCA), they just substitute the NCA propagators into Eq. (9), with the limitations exposed above.

The current-current correlations can be deduced either using Eq. (9) or using the scattering approach for the multiterminal case (see Ref. 24). The latter formalism amounts to replacing the bare parameters by the renormalized ones.\textsuperscript{23} We consider the illustrative case of having different electrochemical potentials in two leads, $\mu_a \neq \mu_b$ (e.g., $\alpha=2$ and $\beta=3$) at zero temperature. We find

\begin{equation}
S_{23}(0) = -\frac{2e^2}{h} \sum_{\gamma,\delta} \int d\epsilon \text{Tr} \langle s_2^\dagger s_2^\dagger s_3^\dagger s_3^\dagger \gamma \delta (f_{\gamma} - f_{\delta})(f_{\delta} - f_{\gamma}) \rangle,
\end{equation}

where $s_{a\beta}$ is the renormalized scattering amplitude of a Breit-Wigner resonance

\begin{equation}
s_{a\beta}(\epsilon) = \delta_{a\beta} - \frac{2i \sqrt{\Gamma_{a\alpha}^{\ast} \Gamma_{\bar{\beta} \bar{\bar{\beta}}}}}{\epsilon - \epsilon_{\alpha} + i \Gamma_{\sigma}}.
\end{equation}

In Eq. (11) the trace $\text{Tr}(\ldots)$ is over spin indices. The Fermi functions $f_a$ and $f_b$ are arbitrary.\textsuperscript{24} Choosing $f_a=f_b=f_3$, we obtain

\begin{equation}
S_{23}(0) = -\frac{2e^2}{h} \sum_{\sigma} \int d\epsilon \text{Tr} \langle \tilde{T}_{12}^{\sigma} \tilde{T}_{13}^{\sigma} (f_1 - f_3)^2 + \tilde{R}_{23}^{\sigma} \tilde{T}_{13}^{\sigma} (f_2 - f_3)^2 \rangle
\end{equation}

\begin{equation}
+ 2 \tilde{T}_{12}^{\sigma} \tilde{T}_{23}^{\sigma} (f_1 - f_3)(f_2 - f_3),
\end{equation}

where $R_{23}^{\sigma}$ is the reflection probability (in general $R_{aa} = 1 - \Sigma_{\beta}^{\beta} T_{ab}$). Notice that, generally, one cannot write the multilead current-current correlations only in terms of transmission probabilities as in Eq. (13). This was pointed out by Büttiker,\textsuperscript{44} suggesting the appearance of exchange effects in noise measurements. Here, because we are dealing with a (renormalized) Breit-Wigner resonance, exchange corrections due to phase differences do not play any role.

For completeness, we give now the formula for the shot noise, i.e., the current-current correlations measured at the same lead (e.g., lead 1). Following the way of reasoning that led to Eq. (13) we obtain

\begin{equation}
S_{11}(0) = \frac{2e^2}{h} \sum_{\sigma} \int d\epsilon \text{Tr} \langle \tilde{T}_{12}^{\sigma} \tilde{R}_{11}^{\sigma} (f_1 - f_2)^2 + \tilde{T}_{13}^{\sigma} \tilde{R}_{11}^{\sigma} (f_1 - f_3)^2 \rangle
\end{equation}

\begin{equation}
+ \tilde{T}_{12}^{\sigma} \tilde{T}_{23}^{\sigma} (f_2 - f_3)^2.
\end{equation}

III. RESULTS

In the following, we present results obtained by self-consistently solving Eqs. (4) and (5) for each bias voltage. The rest of the parameters is changed in Sec. III A–III E. Throughout this work, we have checked that current conservation ($I_1 + I_2 + I_3 = 0$) is fulfilled.\textsuperscript{45}

Tunneling effects are incorporated at all orders since at equilibrium the Kondo temperature is found to be

\begin{equation}
k_B T_K = D \exp(-\pi |\epsilon_0|/2|\Gamma|),
\end{equation}

which is clearly a nonperturbative result. In Eq. (15) $\Gamma = \sum_{\alpha=1}^{\alpha} \Gamma_\alpha$ is the total hybridization broadening. The reference energy will be always set at $E_F=0$ and the energy cutoff is $D=100\Gamma$. The bare level is $\epsilon_0=-6\Gamma$, deep below $E_F$ to ensure a pure Kondo regime.

A. Dephasing

Before turning to the determination of current cross correlators, we briefly discuss with an application the capabilities of \textit{three-terminal} setups to illustrate some difficult aspects of the physics of the \textit{two-terminal} Kondo effect. As mentioned in the Introduction, we investigate the action of a fictitious voltage probe\textsuperscript{46} (say, lead 3) in order to \textit{simulate} decoherence effects on the formation of the Kondo resonance between leads 1 and 2.\textsuperscript{47} These contacts play the role of source and drain, respectively. The voltage probe model\textsuperscript{46} describes de-
coherence because an electron that is absorbed into the probe looses its coherence. The exiting electron is replaced by an electron (with an unrelated phase) injected by the probe.

At low temperatures the principal source of dephasing is due to quasielastic scattering. We consider then a voltage probe that preserves energy. The current through the voltage probe is zero at every energy. 49 The current through the voltage probe depends on the coupling to the probe. At each bias, the conductance suppression depends on the coupling to the probe. The degree of the suppression level of the Kondo resonance is a function of the bare coupling \( \Gamma_1 \) to the voltage probe (reservoir 3) for \( \Gamma_1 = \Gamma_2 \) and \( \epsilon_0 = -6\Gamma \). (b) Linear conductance \( G_{11}(0) \) showing the reduction of the peak in (a) vs the coupling to the voltage probe. The dots are numerical results where the line corresponds to an analytical formula (see text).

FIG. 2. (a) Differential conductance \( G_{11} \) vs bias voltage \( V_1 \) as a function of the bare coupling \( \Gamma_3 \) to the voltage probe (reservoir 3) for \( \Gamma_1 = \Gamma_2 \) and \( \epsilon_0 = -6\Gamma \). (b) Linear conductance \( G_{11}(0) \) showing the reduction of the peak in (a) vs the coupling to the voltage probe. The dots are numerical results where the line corresponds to an analytical formula (see text).

We have to insert this result into Eqs. (4) and (5) and solve self-consistently for the hybridization couplings \( \Gamma \) and the resonance level \( \tilde{\epsilon}_0 \) in the presence of quasi-elastic scattering for each value of the applied bias voltage. Then we compute numerically the differential conductance \( G = dI/dV_{sd} \), where \( I = I_1 = I_2 \) and \( V_{sd} = V_1 - V_2 \).

FIG. 3. (a) Differential conductance \( G_{11} \) vs bias voltage \( V_1 \) for \( \epsilon_0 = -6\Gamma \) (\( T_h^0 = 8 \times 10^{-3} \Gamma \)). (b) Dependence of the Kondo temperature on \( V_1 \).

At low temperatures the principal source of dephasing is due to quasielastic scattering. We consider then a voltage probe that preserves energy. The current through the voltage probe is zero at every energy. Thus, from Eq. (7) the distribution function at the probe reads

\[
f_3(\epsilon) = \frac{T_{13}(\epsilon)f_1(\epsilon) + T_{23}(\epsilon)f_2(\epsilon)}{T_{13}(\epsilon) + T_{23}(\epsilon)}.
\]  

We have to insert this result into Eqs. (4) and (5) and solve self-consistently for the hybridization couplings \( \Gamma \) and the resonance level \( \tilde{\epsilon}_0 \) in the presence of quasielastic scattering for each value of the applied bias voltage. Then we compute numerically the differential conductance \( G = dI/dV_{sd} \), where \( I = I_1 = I_2 \) and \( V_{sd} = V_1 - V_2 \).

From now on, we consider lead 3 as a real electrode with tunable voltage \( V_3 \). We set \( V_2 = 0 \) and vary the tunneling coupling \( \Gamma_3 \). The self-consistent results of Eqs. (4) and (5) are inserted in Eq. (7) to calculate the differential conductance \( G_{11} \) as a function of \( V_1 \). At \( \Gamma_3 = 0 \) the conductance at \( V_1 = 0 \) achieves the unitary limit as in the two-terminal case. With increasing the coupling to third lead, \( G_{11}(0) \) decreases. For equal tunnel couplings \( \Gamma_1 = \Gamma_2 = \Gamma_3 = \Gamma / 2 \), \( G_{11}(0) \) does not reach 1 (in units of \( 2e^2/h \)) but instead \( G_{11}(0) = 8/9 \), in agreement with Ref. 51. This is an immediate consequence of having three leads with identical couplings. Interestingly, the Kondo temperature of Fig. 3(b) does not vanish abruptly for \( V_1 = 2T_h^0 \) as known in the two-terminal case (see the case \( \Gamma_3 = 0 \)). This is an important result as it implies that Kondo correlations survive at large voltages. The effect is reminiscent of the situation found by Aguado and Langreth in tunnel-coupled double quantum dots, though the physical origin is clearly distinct.

B. Multiterminal conductance

C. Sign of current cross correlations: Comparison with a noninteracting quantum dot

We now focus on the current-current correlations of the current for \( V_2 = 0 \) and equal couplings \( \Gamma_1 = \Gamma_2 = \Gamma_3 = \Gamma / 3 \). Later, we shall allow for nonzero voltage differences between leads 2 and 3. In Fig. 4(a), we show the cross correlator \( S_{23}(0) \) obtained from Eq. (13). As expected, \( S_{23} \) is zero for \( V_1 = 0 \) and negative elsewhere. This reflects the fermionic nature of the quasiparticles. For comparison, we plot in Fig. 4(b) the corresponding \( S_{23} \) for a noninteracting resonant double-barrier structure with the level at \( E_F \) (of course, for \( \epsilon_0 = -6\Gamma \) the spectrum \( S_{23} \) is always very small, as is the
transmission). In this case, the physics is governed by the bare coupling $\Gamma$ (Ref. 52). On the contrary, in the Kondo problem the dominating energy scale is $T_K$. Qualitatively, Figs. 4(a) and 4(b) look the same until $V_1 - 2T_K$. The cross correlator in the Kondo case increases with voltage while in the noninteracting case $S_{23}$ saturates at large voltages. It is easy to show that the saturation value is given by $-8\pi/81 = -0.31$ (in units of $4e^2/\hbar$). The reason for the increase of $S_{23}(0)$ in Fig. 4(a) is that $T_K$ is voltage dependent unlike the bare $\Gamma$, even in the wideband limit. In particular, the current-voltage characteristics shows a region of negative differential conductance in the Kondo case [see Fig. 3(a)] whereas it reaches a constant value at large voltages for a noninteracting quantum dot.

To avoid effects due to moderate biases, in what follows we shall concentrate on a normalized $S_{23}$. We define the Fano factor of $S_{23}$ as

$$\gamma_{23} = \frac{S_{23}}{2\varepsilon |I_2||I_3|}.$$  

If the scattering region were a simple barrier of transmission $T$, $\gamma_{23}$ would be simply $-1$. This number changes when the system under consideration is a quantum dot. In Figs. 4(a) and 4(b), we plot $S_{23}$ for the Kondo and the noninteracting case, respectively. Their corresponding Fano factors are shown in Figs. 4(c) and 4(d). We see that $\gamma_{23}$ has a minimum at $V_1 = 0$. Analytically, we find $\gamma_{23}(0) = -4/9 \approx -0.44$, which is in excellent agreement with the numerical result. Likewise, we can assess the limit of $\gamma_{23}$ at very high voltages ($V_1 \gg T_K^0$). We get $\gamma_{23} = -2/9 \approx -0.22$. As observed, both curves tend to this value, though for a noninteracting quantum dot it is more quickly due to the independence of $\Gamma$ on the bias voltage.

### D. Effect of nonequilibrium splitting on current-current correlations

Now we turn to an exciting case. Consider the bias configuration $V_2 = -V_3 \neq 0$ and determine the differential conductance $G_{11}$ as a function of $V_1$. The case $V_2 = -V_3 = 0$ has been treated before. However, due to the fact that the boson field never vanishes, we can study the situation $\Delta V = |V_2 - V_3| \approx 2T_K^0$. As remarked in the Introduction, it has been argued \(^{11,12}\) and experimentally observed \(^{13}\) that in a three-lead geometry the splitting of the Kondo resonance due to voltage is visible, unlike the two-terminal case. Moreover, in Refs. 12 and 51 it has been noted that the conductance $G_{11}$ is not sensitive to the strength of the coupling to the third lead, showing always a two-peak structure. Of course, only when the third lead is weakly coupled to the dot $G_{11}$ is a measure of the LDOS. But since we are interested in the transport properties of the system, our choice of equal coupling constants does not affect the results for the conductances and the current-current correlations.

In Fig. 5 we plot the behavior of the differential conductance $G_{11}$. At $\Delta V = 0$ we obtain the zero-bias anomaly of Fig. 3(a). As $\Delta V$ increases, $G_{11}$ is split at $V_2 \approx -T_K^0$. Both splitting peaks lie at $V_1 - V_3$ and $V_1 - V_2$, i.e., when a pair of electrochemical potentials are aligned. It is also at those points where the Kondo temperature is larger. We emphasize that this effect has no similitude in the electronic transport through a noninteracting quantum dot. Still, a mean-field theory of the Kondo effect as presented here is able to capture this physics. At the same time that the splitting in $G_{11}$ develops, the height of the peaks decreases, suppressing the zero-bias anomaly, although not so strongly as in the experiment \(^{13}\) due to the absence of inelastic scattering in this case.

We now use Eq. (13) to calculate the cross correlations between leads 2 and 3. The results are presented in Fig. 6(a). The dependence of $S_{23}$ on voltage is rather asymmetric, hindering the observation of a clear indication due to the voltage induced splitting. The asymmetry is caused by the third term of the right-hand side of Eq. (13), which is not symmetric under the operation $V_1 \rightarrow -V_1$ when $\Delta V > 0$. That is the reason why we next consider the shot noise in lead 1 $S_{11}$, which is an even function of the applied $V_1$.

In Fig. 6(b) we plot the results of Eq. (14). We observe that $S_{11}$ at $V_1 = 0$ is nonvanishing with increasing $\Delta V$, causing a divergence of the Fano factor. This is not related to the Kondo physics but the lead 1 at $V_1 = 0$ acts as a voltage probe with zero impedance since the net current flowing through it is zero. Including the fluctuations of the potentials would probably cancel out the divergence. A consequence of Kondo
physic...ts of the cross correlator at different lead magnetization when $V_2 = V_3 = 0$. (a) Parallel alignment between the magnetizations of the leads with spin polarizations: $p_1 = p_2 = p_3 = p$. (b) Antiparallel case with $p_1 = -p_2 = -p_3 = p$.}

**E. Spin-dependent transport and current cross correlations**

So far we have assumed spin-independent transport. Let us go back to the bias configuration of Secs. III B and III C ($V_2 = V_3 = 0$) and focus on the spin-dependent transport properties. It is customary in the theoretical studies of spintronic transport to take into account the influence of external magnetic fields and ferromagnetic electrodes, among other parameters. First, we shall change the external Zeeman field and then enable the presence of spin-polarized tunneling.

**1. Magnetic field**

We assume that the leads are paramagnetic and that the magnetic field is applied only to the dot, resulting in a Zeeman gap of the bare resonant level: $\Delta_2 = e_0 - e_0$. It is well known that, as a consequence, the Kondo resonance is split when $\Delta_2 \sim T_K^0$. Figure 7(a) shows the differential conductance $G_{11}$ for different values of the Zeeman field. The conductance is split and quenched with increasing $\Delta_2$, as expected. In Fig. 7(b), we depict the Fano factor of the cross correlator $\gamma_{23}$. It exhibits a very interesting feature. Due to the splitting of the Kondo peak, the minimum of the cross correlator at $V_1 = 0$ becomes a local maximum, resulting from the suppression of

**FIG. 8. Fano factor of the cross correlator, $\gamma_{23}$ vs $V_1 / T_K^0$ for different lead magnetization when $V_2 = V_3 = 0$. (a) Parallel alignment between the magnetizations of the leads with spin polarizations: $p_1 = p_2 = p_3 = p$. (b) Antiparallel case with $p_1 = -p_2 = -p_3 = p$.**

the Kondo effect. However, this change occurs before the splitting of the conductance $G_{11}$. Therefore, measuring the shot noise provides additional information in this case. The presence of the splitting would be detected in an experiment more precisely by means of the shot noise. The underlying reason is that the form of Eq. (13) differs from that of the current, which is basically proportional to $\tilde{T}_{12}$ alone, see Eq. (7). As a result, the width of the $G_{11}$ resonance is a bit larger than the $\gamma_{23}$ antiresonance and the former is then more robust than the latter against the application of magnetic fields.

**2. Ferromagnetic leads**

There has recently been considerable debate about the influence of ferromagnetic leads in the Kondo physics of a quantum dot. In Sec.III E 1, it was clear that an external magnetic field alters the real part of the quantum-dot self-energy, breaking the spin degeneracy. In the case of spin polarized tunneling, the situation is more subtle. When the magnetic moments of the contacts are aligned along the same direction, the density of states of the localized electron undergoes a splitting if particle-hole symmetry is broken. Recent transport experiments with C_{60} molecules and carbon nanotubes have addressed this regime. However, in our case the dot is in the strong coupling limit and the Kondo effect is pure in the sense that no charge fluctuations are allowed. Thus, no splitting is expected in the differential conductance.

In Fig. 8(a), we show the cross correlator $\gamma_{23}$ for different values of the lead magnetization in the parallel case. This means that $p_1 = p_2 = p_3 = p$, where $p_\alpha$ is the spin polarization of lead $\alpha$. Ferromagnetism in the leads arises through spin-dependent densities of states $\nu_{\sigma\alpha}(\epsilon) = \sum_\delta \delta(\epsilon - e_{\sigma\alpha})$. Hence, the linewidths become spin dependent: $\Gamma_{\sigma\alpha} = (1 / p_\alpha) \Gamma_{\sigma\alpha}$, where $+(-)$ corresponds to up- and (down)spins. We prefer to restrict $p_\alpha$ to small values as strong magnetizations would require a proper treatment of the reduction of the bandwidth $D$. We observe that $\gamma_{23}$ is rather insensitive to changes in $p$ in the same fashion as $G_{11}$ is in the Fermi-liquid fixed point. Only at moderate polarizations ($p = 0.6$) we see that the dip in $\gamma_{23}$ gets narrower because the Kondo temperature decreases as $p$ increases. In addition, $\gamma_{23}$ is always negative in contrast to the results obtained in the Coulomb blockade regime, where $\gamma_{23}$ can take positive values.  

When the spin-
flip scattering rate is smaller than the tunneling rate, $\gamma_{23}$ can be positive. However, in the Kondo regime this condition is never met because the rate of spin-flip scattering $\sim 1/T_K$ is always much larger than the tunneling rate $\sim 1/\Gamma$. Figure 8(b) is devoted to the antiparallel case: $p_1=-p_2=-p_3=p$. Accordingly, $\gamma_{23}$ is lifted with increasing lead polarization because the conductance peak decreases with increasing $p$ (roughly, with a factor $1-p^2$).

IV. CONCLUSION

In summary, we have investigated the Kondo temperature, the differential conductance, and cross correlations of the current when three leads are coupled to an artificial Kondo impurity in the Fermi-liquid fixed point of the infinite-U Anderson Hamiltonian $(T\ll T_K)$. We have performed a systematic study of the properties of the cross correlators when $eV$, Zeeman splittings, and ferromagnetic leads influence the nonequilibrium transport through the quantum dot. Our most relevant result is the behavior of the shot noise when there arises a voltage-induced splitting in the quantum dot.

In addition, we have studied the current of a two-terminal quantum dot attached to a voltage probe. We have shown that increasing the coupling with the probe induces a quenching of the Kondo peak. Despite the simplicity of this approach, it gives rise to results that are in agreement with more sophisticated models, though the precise processes responsible for the decoherence still need to be derived from a microscopic model.

We have not exhausted all the possibilities that the model offers and more complicated geometries with appealing results can be envisaged. One could address the situation with two injecting and two receiving leads, which could give rise to Hanbury Brown–Twiss-like effects. We expect that phase related exchange terms will arise especially at higher temperatures ($T>T_K$), when the singlet state between the localized spin and the conduction electrons is not yet well formed. We believe that in the presence of spin-polarized couplings due to ferromagnetic leads, bunching effects will be enhanced.

Improvements of the model should go in the direction of including fluctuations of the boson field and of the renormalized level. However, we do not expect large deviations from the results reported here when $T\ll T_K$. These fluctuations will evidently become important as temperature approaches $T_K$. Experimentally, our predictions can be tested with present technology, such as GaAs quantum dots or carbon nanotube nanostructures.

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THREE-TERMINAL TRANSPORT THROUGH A QUANTUM...

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